Dry Sand, Wet Sand

Digging into the physics of sandpiles and sand castles

By IVARS PETERSON

In *Les Misèrables*, novelist Victor Hugo wrote, "Who could ever calculate the path of a molecule? How do we know that the creations of worlds are not determined by falling grains of sand?"

The movements of grains of sand have played a significant part in shaping the natural world. Granular materials, including sand, are also ubiquitous in our everyday lives. From powders to gravel, such materials are of vital importance in a wide range of industries, including agriculture, mining, construction, food processing, and pharmaceuticals.

Moreover, what happens in a pile of sand on a tabletop, whether used as metaphor or model, turns out to be relevant to a wide range of physical processes.

As recent research has demonstrated, understanding sand and its kin—even when at rest—poses a multitude of scientific challenges. Tiny effects such as the placement of individual grains and their precise motion can have significant consequences.

Granular materials display a curious blend of properties. Dry sand, for instance, can be poured like water. Unlike a liquid, however, it can also support the weight of a person strolling along a beach. And as just about any child knows, a little bit of moisture can turn sand into a remarkable construction material.

Over the years, engineers have characterized and studied many different granular materials, ranging from sand to coal dust. During the last decade, physicists have entered the picture, attempting to elucidate the basic mechanisms that underlie the behavior of these pourable solids (SN: 7/15/89, p. 40).

"Even a small improvement in our understanding of granular media could have a profound impact on industry," contended physicists Heinrich M. Jaeger and Sidney R. Nagel of the University of Chicago and Robert P. Behringer of Duke University in Durham, N.C., in the October 1996 Reviews of Modern Physics.

Even in a resting state, granular materials exhibit unusual behavior. One might think, for example, that being buried under 30 meters of sand would be more oppressive than lying only 3 meters below the surface. That's not necessarily the case, however.

When a liquid occupies a tall cylinder, the pressure the liquid exerts at the bottom increases in direct proportion to the liquid's height. For a granular material, the pressure at the base doesn't increase indefinitely as the material's height increases; instead, it reaches a maximum value and stays there.

Indeed, it is this characteristic that allows sand to trickle at a nearly constant rate through the narrow opening separating the two glass bulbs of an hourglass, making it a handy instrument for measuring elapsed time.

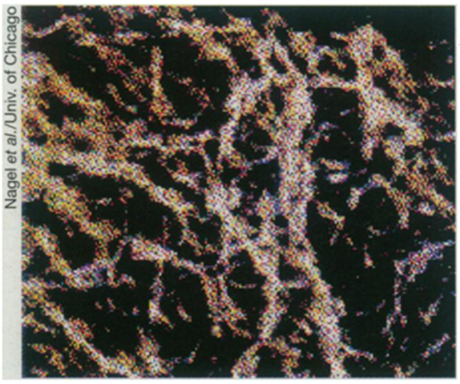

Nagel et al./Univ. of Chicago

Image of the force chains produced in a box of glass beads under pressure. Stressed beads appear as bright spots.

Contact forces between grains in a pile transfer weight to the container's walls, which bear the extra load. To visualize how the force of gravity acting on a column of a granular material is transmitted downward and outward by the grains, Nagel and his coworkers studied the behavior of glass beads immersed in a mixture of water and glycerol. Because each bead rotates polarized light in proportion to the amount of stress it experiences, the researchers could see where within the material the forces are greatest.

They found that the forces are not distributed evenly throughout the material. The column's weight is carried from grain to grain along jagged chains—a network of lightning bolts of concentrated force. As a result, the container's walls rather than its base carry much of the weight. Moreover, the force is significantly larger at some points of contact than at others.

The presence of what Nagel and his colleagues call force chains may account for the sudden rupture of the side walls of a silo, when grain structures happen to deliver an enormous force to a particular weak spot. In contrast, the uniform force exerted by a liquid at that level wouldn't be as likely to cause a break.

To measure the forces that press on a container's bottom, Nagel and his team placed a piece of carbon paper under the beads. The size of the smudge left by an individual bead provided an estimate of the proportion of the total weight supported by that bead. The result showed an uneven distribution of loads.

To help interpret the patterns produced in these experiments, Chicago's Susan N. Coppersmith and her colleagues devised a simple theoretical model in which they assumed that a given bead transmits the load it bears unequally and randomly onto the three beads on which it rests. Using the model, they characterized the expected statistics of the force fluctuations from place to place within a granular material and obtained good agreement with experimental results.

This model, however, doesn't capture the full three-dimensional nature of the forces acting between grains, in which the downward force of gravity is translated into the sum of forces acting in different directions between the grains, depending on their angle of contact.

Incorporating these so-called vector forces has proved immensely difficult. Researchers have found it especially tricky to formulate equations embodying the observation that the particles rest against each other without sticking. "It's clear that the lack of cohesion is important, but we don't know yet how to handle that theoretically," Coppersmith says.

Dumped on a table, dry sand settles into a cone-shaped mound. Interestingly, experiments have shown that the pressure exerted by a conical pile on a tabletop is highest not in the middle, directly under the peak, but closer to the edge.

Theorists have proposed a number of explanations for this curious pressure distribution. In developing their models, they have encountered the same problem that Coppersmith and her colleagues faced in accounting properly for the three-dimensional forces between grains.

Basically, "you have too many unknowns and not enough equations," Coppersmith says.

Last year, Michael E. Cates of the University of Edinburgh and his collaborators proposed a theoretical approach incorporating the notion that grains tend to settle into a pattern of "nested arches," an assortment of bridges oriented in all directions. Such arches would do the same sort of job as the arches that support a cathedral's ceiling. Arches of sand grains steer the pile's weight away from the center, producing the observed central pressure dip.

By assuming that the forces are transmitted along certain directions defined by a system of nested arches, the researchers obtained a complete set of equations that allowed them to calculate the distribution of pressure across the base of a pile. Their calculations showed reasonably good agreement with experimental data.

Cates and his coworkers acknowledge that their approach includes mathematical relations that depend on details of how the pile was constructed—whether it was built up one dropped grain at a time or simply dumped all at once. In their model, each small cluster of grains inside a pile has to "remember" how it got there, and the orientation of the forces between grains must remain unchanged as more grains pile up above them. Physicists don't yet know the basis for such memory.

In fact, several complications make the pressure dip phenomenon hard to understand. Experimental results are very sensitive to small details, such as the slight buckling of the surface on which the pile rests. Moreover, recent computer simulations and calculations suggest that at least two additional mechanisms of force transmission between grains may contribute to the effect.

What may eventually happen is a coming together of force chains and arches into a new theoretical picture that integrates local fluctuations with overall structure, Coppersmith says.

For many players, the computer game Tetris has become a highly addictive pastime. Among physicists, it has also inspired a simple geometric model of the settling process in dry sand and other granular materials.

One can imagine the grains in a silo, for example, sitting like a stack of untidily arranged bricks. The fraction of the volume actually occupied by the grains can vary greatly, depending on how the container was filled. Vibrations cause settling, decreasing the total volume and increasing the packing density.

To investigate geometric aspects of granular packing, Hans J. Herrmann of the Institute for Computer Applications at the University of Stuttgart in Germany and his collaborators have developed a two-dimensional, Tetris-like model in which rectangular particles with different orientations drift downward and settle into place, filling a cylinder. They describe the model in the Aug. 25 Physical Review Letters.

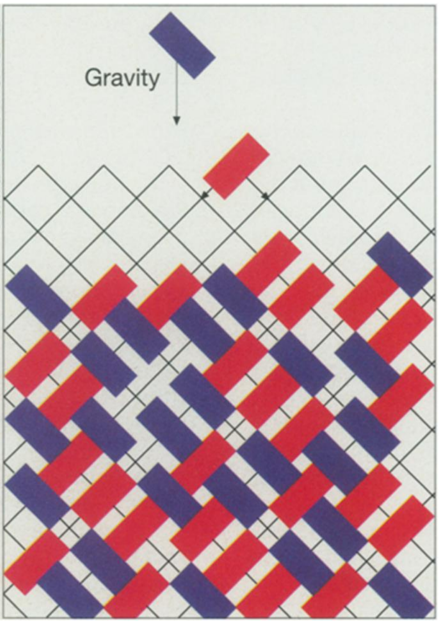
In Tetris, as variously shaped blocks drop downward on the computer screen, a player nimbly taps in keyboard commands to rotate the falling units or shift them to the left or right. When the blocks reach the bottom, they pile up. The tidiness of the arrangement reflects the skill of the player.

The key idea that the model encapsulates is that the orientations of adjacent particles tend to lock them into certain positions—an effect known as geometric frustration—from which progress to a lower energy arrangement is prevented. However, those positions can change abruptly when a vibration shakes the entire assemblage, leading to compaction.

Geometric frustration is important because it underlies the formation of cavities and arches within a granular material, Herrmann says. It must also be considered in describing how a collection of grains shifts from one configuration to another.

The general question of jamming, in which any system—whether sand, glass, or foam—is frustrated, is the topic of a program organized by Nagel and being held this month at the Institute for Theoretical Physics at the University of California, Santa Barbara. Such jamming can occur because of the geometry of the medium or other factors.

"My aim in putting this together was to try to see what similarities and differences there are in jammed states in different systems," Nagel says. In the case of granular materials, "how does the process by which you bring the granular material to a static state relate to the properties of that state?"



Herrmann *et al.*/Phys. Rev. Lett.

Diagram of a geometric configuration involving differently oriented particles in a Tetris-like model of a granular material.

Constructing a sand castle provides a striking demonstration of the tremendous difference between the physical properties of wet and dry sand. Wet sand grains tend to clump together, showing a cohesion entirely absent in dry sand.

The key factor is surface tension. Films of water surrounding each grain form bridges that link the particles and hold them together. What's surprising is how little liquid is necessary to achieve the effect.

In the June 19 Nature, physicists Peter Schiffer, Albert-László Barabási, and their coworkers at the University of Notre Dame in Indiana describe an experiment in which they added small amounts of light oil to polystyrene beads and determined the mixture's angle of repose—the steepest stable slope that a pile can exhibit. They discovered that a liquid layer only 50 nanometers thick, coating beads about 0.8 millimeter in diameter, was sufficient to produce strong cohesion.

"The amounts of liquid that we added were incredibly small, but we still got a big effect," Barabási says. Previous engineering studies of wet granular materials, such as coal, sugar, seeds, or rock chips, had typically focused on mixtures with a much higher liquid content or on particular materials that are sometimes also porous or soluble in water.

"Rather than trying to understand a system of practical interest, we're trying to understand the basic physics," Schiffer says. So "we kept our system simple, and we carefully controlled the amount of liquid." The use of oil, for example, greatly reduced evaporation and circumvented the potentially confounding effect of humidity.

The Notre Dame team is preparing to study how the properties of sand vary as the amount of liquid changes. "Sand pours like a liquid when it's dry," Barabási says. "It acts like a liquid again when there's much more liquid than sand. In between, sand is structurally stable, almost like a solid. Our interest is in studying that transition."

"The results from Notre Dame are preliminary and pose more questions than they answer," Herrmann comments. "But they are interesting, and certainly more work in that direction will be done."

"Eventually, we'd like to look at more realistic systems—the sorts of things that geologists or industrial engineers would be interested in," Schiffer says.

There is much to be learned from digging into sandpiles. □