Prize offered for solving number conundrum

For more than 300 years, the mathematical problem known as Fermat's last theorem attracted a plethora of would-be conquerors. The theorem was finally proved just a few years ago by Andrew Wiles of Princeton University as part of a larger effort that illuminates links between number theory and geometry (SN: 11/5/94, p. 295).

The complicated, lengthy proof that Wiles found hasn't satisfied everyone, however. Some still wonder whether a simpler proof could be found—one that Pierre de Fermat himself may have had in mind but failed to write down centuries ago.

"The mystery remains: Is there an elementary proof?" asks Andrew Beal, a banker in Dallas and an amateur mathematician interested in number theory.

In grappling with the problem, Beal has looked for solvable equations analogous to the one expressing Fermat's last theorem. "I would try to understand why the solutions of the new equation can't be converted to solutions of the Fermat equation," he says.

As a result of those studies, Beal has formulated a conjecture involving equations of the form $A^x + B^y = C^z$. The six letters represent whole numbers, with x, y, and z greater than 2. Fermat's last theorem involves the special case in which the exponents x, y, and z are the same.

Beal noticed that when a solution of the general equation existed, A, B, and C had a common factor. For example, in the equation $3^6 + 18^3 = 3^8$, the numbers 3, 18, and 3 all have the factor 3.

Using computers at his bank, Beal checked equations with exponents up to 100. "I couldn't come up with a solution that didn't involve a common factor," he says. The question is whether that is always true.

Beal has now offered a prize of $50,000 to anyone who can prove the conjecture or $10,000 to anyone who can find a counterexample.

Interestingly, Beal's conjecture is closely related to questions of considerable concern to number theorists. "It is remarkable that occasionally someone working in isolation and with no connections to the mathematical [community] formulates a problem so close to current research activity," R. Daniel Mauldin of the University of North Texas in Denton comments in the December NOTICES OF THE AMERICAN MATHEMATICAL SOCIETY, where the Beal prize problem is announced. Mauldin heads the prize committee and will handle all inquiries and proposed proofs.

Whether a proof is likely to materialize anytime soon isn't evident at the moment. "It's not clear how you would go about solving it or whether the methods of Wiles could be extended this far," says Andrew J. Granville of the University of Georgia in Athens.

Intriguingly, introducing a coefficient larger than 1, so that the equation reads, say, $A^x + 31B^y = C^z$, allows all sorts of possible solutions when x, y, and z are greater than 2. In the Beal conjecture, the coefficients are all 1, which appears to put the resulting equations and their solutions in a special category.

The deeper question is one of finding a general method of identifying all solutions to the equation, whether there are just a few or an infinite number of them, Granville says. "That would be a wonderful advance." —*I. Peterson*

Holey device traps light for lasers, filters

Structures known as photonic crystals block the transmission of light whose wavelength falls within a certain range. Researchers from the Massachusetts Institute of Technology have now built a silicon device that integrates a photonic crystal with a light conduit, making a component that could lead to more efficient lasers and better optical filters.

The new device consists of a thin strip of silicon—the conduit—deposited on silicon dioxide, with a row of eight holes, each 0.2 micrometer (μm) in diameter, "punched all the way through," says MIT's Pierre R. Villeneuve. The holes form the photonic crystal, blocking out a 400-nanometer swath of wavelengths in the infrared part of the spectrum (SN: 3/28/92, p. 206).

The holes are spaced at 0.22 μm intervals, except for the fourth and fifth ones, which are farther apart. This irregularity, essentially a defect in the crystal, captures one wavelength of light that would otherwise get blocked. The holes on either side of the larger gap act as mirrors, bouncing the light back and forth until it emerges from the silicon as a strong signal at 1.54 μm. The device can thus emit light of a chosen frequency or act as a switch that picks out that frequency from a broader signal.

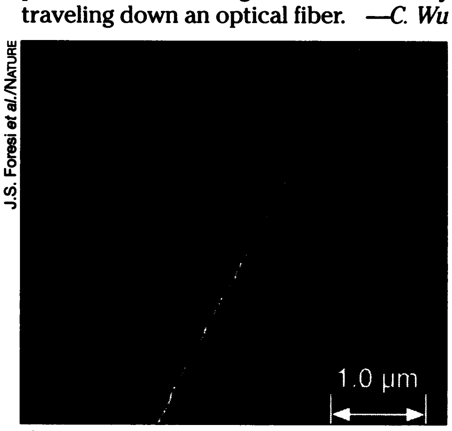
Villeneuve and his colleagues report their findings in the Nov. 13 NATURE.

Adjusting the number, size, and spacing of the holes affects the wavelength and intensity of the emitted signal and the range of the blocked wavelength. Photonic crystals that block infrared and visible light are difficult to make because of the small feature size required. The MIT group used a technique called X-ray lithography, which is not yet widespread in industry (SN: 11/8/97, p. 302).

MIT's achievement represents "a step on the way to making very tiny [components] from other materials" that have better optical properties than silicon, says Eli Yablonovitch of the University of California, Los Angeles.

Lasers and light-emitting diodes made with photonic crystals would operate with much lower currents than conventional devices, Villeneuve says. In telecommunications, such crystals could pick out individual signals from the many traveling down an optical fiber. —*C. Wu*

J.S. Foresi *et al.*/NATURE



A strip of silicon with eight holes acts as an optical filter. Infrared light gets trapped and amplified in the gap between the fourth and fifth holes.

Biologists peck at bird-dinosaur link

In the paleontological paternity suit of the century, researchers are battling over the widely held idea that dinosaurs sired birds. A small but vocal group of naysayers has scored points recently with two separate studies arguing that the lungs and hands of birds were starkly different from those of theropod dinosaurs—the small, bipedal carnivores that supposedly evolved feathers and ultimately flight.

"Our work strongly suggests that dinosaurs may not have been the direct ancestor of birds," says John A. Ruben, a physiologist at Oregon State University in Corvallis. In the Nov. 14 SCIENCE, Ruben and his colleagues compared the lungs of living birds and crocodiles with evidence from dinosaur fossils.

Crocodiles, like mammals, draw air into their lungs with the help of a diaphragm, a movable tissue that divides the chest cavity from the abdomen. In crocodiles, muscles from the pubic bone in the pelvis attach to the liver, which connects to the diaphragm, creating a pistonlike system for pulling the diaphragm toward the animal's tail. When these muscles contract, the movement of the diaphragm enlarges the chest cavity and pulls air into the lungs.

Birds have a different lung system, one that doesn't rely on a diaphragm to alter the pressure in the chest cavity. Instead,

Continued next page.