Prize offered for solving number conundrum

For more than 300 years, the mathematical problem known as Fermat's last theorem attracted a plethora of would-be conquerors. The theorem was finally proved just a few years ago by Andrew Wiles of Princeton University as part of a larger effort that illuminates links between number theory and geometry (SN: 11/5/94, p. 295).

The complicated, lengthy proof that Wiles found hasn't satisfied everyone, however. Some still wonder whether a simpler proof could be found—one that Pierre de Fermat himself may have had in mind but failed to write down centuries ago.

"The mystery remains: Is there an elementary proof?" asks Andrew Beal, a banker in Dallas and an amateur mathematician interested in number theory.

In grappling with the problem, Beal has looked for solvable equations analogous to the one expressing Fermat's last theorem. "I would try to understand why the solutions of the new equation can't be converted to solutions of the Fermat equation," he says.

As a result of those studies, Beal has formulated a conjecture involving equations of the form $A^x + B^y = C^z$. The six letters represent whole numbers, with x, y, and z greater than 2. Fermat's last theorem involves the special case in which the exponents x, y, and z are the same.

Beal noticed that when a solution of the general equation existed, A, B, and C had a common factor. For example, in the equation $3^6 + 18^3 = 3^8$, the numbers 3, 18, and 3 all have the factor 3.

Using computers at his bank, Beal checked equations with exponents up to 100. "I couldn't come up with a solution that didn't involve a common factor," he says. The question is whether that is always true.

Beal has now offered a prize of $50,000 to anyone who can prove the conjecture or $10,000 to anyone who can find a counterexample.

Interestingly, Beal's conjecture is closely related to questions of considerable concern to number theorists. "It is remarkable that occasionally someone working in isolation and with no connections to the mathematical [community] formulates a problem so close to current research activity," R. Daniel Mauldin of the University of North Texas in Denton comments in the December NOTICES OF THE AMERICAN MATHEMATICAL SOCIETY, where the Beal prize problem is announced. Mauldin heads the prize committee and will handle all inquiries and proposed proofs.

Whether a proof is likely to materialize anytime soon isn't evident at the moment. "It's not clear how you would go about solving it or whether the methods of Wiles could be extended this far," says Andrew J. Granville of the University of Georgia in Athens.

Intriguingly, introducing a coefficient larger than 1, so that the equation reads, say, $A^x + 31B^y = C^z$, allows all sorts of possible solutions when x, y, and z are greater than 2. In the Beal conjecture, the coefficients are all 1, which appears to put the resulting equations and their solutions in a special category.

The deeper question is one of finding a general method of identifying all solutions to the equation, whether there are just a few or an infinite number of them, Granville says. "That would be a wonderful advance." —*I. Peterson*

Holey device traps light for lasers, filters

Structures known as photonic crystals block the transmission of light whose wavelength falls within a certain range. Researchers from the Massachusetts Institute of Technology have now built a silicon device that integrates a photonic crystal with a light conduit, making a component that could lead to more efficient lasers and better optical filters.

The new device consists of a thin strip of silicon—the conduit—deposited on silicon dioxide, with a row of eight holes, each 0.2 micrometer (µm) in diameter, "punched all the way through," says MIT's Pierre R. Villeneuve. The holes form the photonic crystal, blocking out a 400-nanometer swath of wavelengths in the infrared part of the spectrum (SN: 3/28/92, p. 206).

The holes are spaced at 0.22 µm intervals, except for the fourth and fifth ones, which are farther apart. This irregularity, essentially a defect in the crystal, captures one wavelength of light that would otherwise get blocked. The holes on either side of the larger gap act as mirrors, bouncing the light back and forth until it emerges from the silicon as a strong signal at 1.54 µm. The device can thus emit light of a chosen frequency or act as a switch that picks out that frequency from a broader signal.

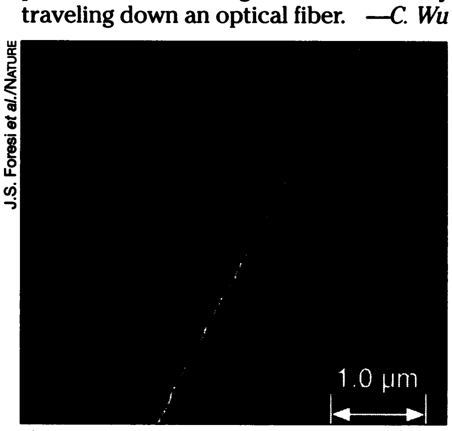
Villeneuve and his colleagues report their findings in the Nov. 13 NATURE.

Adjusting the number, size, and spacing of the holes affects the wavelength and intensity of the emitted signal and the range of the blocked wavelength. Photonic crystals that block infrared and visible light are difficult to make because of the small feature size required. The MIT group used a technique called X-ray lithography, which is not yet widespread in industry (SN: 11/8/97, p. 302).

MIT's achievement represents "a step on the way to making very tiny [components] from other materials" that have better optical properties than silicon, says Eli Yablonovitch of the University of California, Los Angeles.

Lasers and light-emitting diodes made with photonic crystals would operate with much lower currents than conventional devices, Villeneuve says. In telecommunications, such crystals could pick out individual signals from the many traveling down an optical fiber. —*C. Wu*

J.S. Foresi et al./NATURE



A strip of silicon with eight holes acts as an optical filter. Infrared light gets trapped and amplified in the gap between the fourth and fifth holes.

Biologists peck at bird-dinosaur link

In the paleontological paternity suit of the century, researchers are battling over the widely held idea that dinosaurs sired birds. A small but vocal group of naysayers has scored points recently with two separate studies arguing that the lungs and hands of birds were starkly different from those of theropod dinosaurs—the small, bipedal carnivores that supposedly evolved feathers and ultimately flight.

"Our work strongly suggests that dinosaurs may not have been the direct ancestor of birds," says John A. Ruben, a physiologist at Oregon State University in Corvallis. In the Nov. 14 SCIENCE, Ruben and his colleagues compared the lungs of living birds and crocodiles with evidence from dinosaur fossils.

Crocodiles, like mammals, draw air into their lungs with the help of a diaphragm, a movable tissue that divides the chest cavity from the abdomen. In crocodiles, muscles from the pubic bone in the pelvis attach to the liver, which connects to the diaphragm, creating a pistonlike system for pulling the diaphragm toward the animal's tail. When these muscles contract, the movement of the diaphragm enlarges the chest cavity and pulls air into the lungs.

Birds have a different lung system, one that doesn't rely on a diaphragm to alter the pressure in the chest cavity. Instead,

Continued next page.

Two genes help an embryo pick sides

The growing human embryo quickly learns to tell left from right. That's vital, since the outward bilateral symmetry of a human body isn't matched internally. The heart, stomach, and spleen reside on the left, while the liver prefers the right. Even paired organs such as the lungs exhibit some asymmetry; the right lung has three lobes, while the left has two.

To understand how this left-right axis arises, scientists have sought the mutated genes responsible for the rare cases in which the internal organs of people, or mice, are flipped left to right or positioned randomly on either side of the body. They've now found two such genes.

"These are the first demonstrations of single-gene defects that lead to naturally occurring left-right patterning abnormalities in human and mouse," writes Deepak Srivastava of the University of Texas Southwestern Medical Center at Dallas in the November NATURE GENETICS.

Brett Casey of the Baylor College of Medicine in Houston, who describes one of the new genes, grew fascinated with left-right asymmetry many years ago when he performed an autopsy on an infant whose internal organs, particularly the heart, were askew. At the autopsy, a colleague told Casey about a large family in which newborn boys frequently had similar problems.

After studying the family's history, Casey concluded that a mutated gene must exist on the X chromosome. In 1993, he narrowed its location to a small region of the chromosome. In the November NATURE GENETICS, Casey and his colleagues identify the gene and describe mutations found in the original family and in other people with left-right developmental anomalies.

The newfound gene encodes a protein called ZIC3, whose amino acid sequence suggests that it binds to DNA and regulates gene activity. The investigators do not yet know how ZIC3 contributes to formation of left-right asymmetry, but they think its gene acts early in the process.

The discovery of the gene may help unearth additional genes that create the bilateral axis. Mutations in ZIC3's gene account for only a small percentage of people with abnormal left-right asymmetry.

"There are obviously too many genes involved in this complicated cascade to think that any one will be the dominant player," says Casey.

By studying two strains of mice in which some animals are born normal, some with their internal organs inverted completely, and some with organs in random positions, researchers have found a second gene that seems to play a role in left-right asymmetry.

"We think this gene may be involved in the initial determination of left-right asymmetry," says Martina Brueckner of the Yale University School of Medicine.

The gene, described in the Oct. 30 NATURE, encodes a dynein, a protein that forms huge complexes which can serve as molecular motors. Dynein complexes usually interact with long intracellular rods called microtubules, transporting cargo along them or bending them.

Since microtubules have distinguishable ends and dyneins always move toward the same end, the microtubules may act as one-way streets that establish an original left-right asymmetry.

"You've got to have something asymmetric to create new asymmetry. You can't create it out of nothing," Brueckner notes. "We're assuming that the original asymmetric molecule is the microtubule and that the dynein takes that asymmetry and converts it into cellular asymmetry."

Determining how that conversion takes place is next on the agenda, she adds. —*J. Travis*

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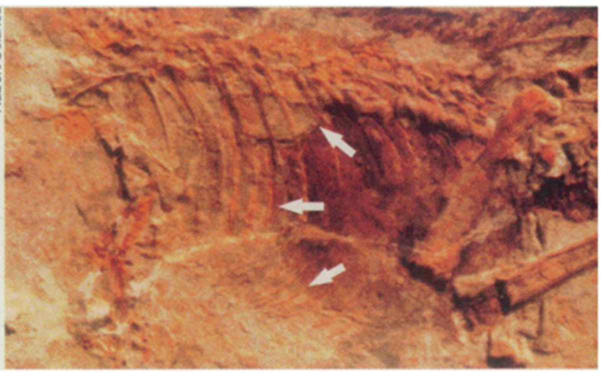
muscles connected to the ribs draw air into a network of sacs located in the abdomen—an extremely efficient system for supplying the oxygen needed during flight.

Ruben's team contends that the pubic bones of theropod fossils closely resemble those of crocodiles, providing evidence that dinosaurs had a piston-driven diaphragm.

They gleaned additional clues about the breathing mechanism of dinosaurs from a Chinese dinosaur called *Sinosauropteryx* (SN: 5/3/97, p. 271). One fossil of this animal preserves evidence of internal organs that normally don't show up in dinosaur remains. These soft-tissue impressions in the chest cavity of *Sinosauropteryx* match the shape and placement of the diaphragm in crocodiles, offering further hints that dinosaurs breathed like crocodiles, says Ruben.

He and his colleagues argue that birds could not have evolved from animals equipped with diaphragms because the two breathing systems work in incompatible ways.

That conclusion supports the findings of another study, published in the Oct. 24 SCIENCE, which focuses on the hand bones of birds and dinosaurs. All land-dwelling vertebrates—from swallows to snakes—descended from ancestors that had five digits on each limb. Most theropod dinosaurs lost two of the digits on their hands during evolution, a pattern broadly similar to the three digits in the wings of birds.



Ruben/SCIENCE

The chest of a dinosaur called Sinosauropteryx *may hold evidence of an internal diaphragm (arrows) used in breathing.*

Ann C. Burke and Alan Feduccia of the University of North Carolina at Chapel Hill contend, however, that the resemblance between bird wings and dinosaur hands is superficial. The researchers studied the embryos of birds, turtles, and alligators to decipher how the digits of the hands grow.

They observed that the fourth digit develops first in all modern vertebrate embryos that have five digits. This finding enabled them to number the developing digits in bird embryos. They determined that birds lack the first and fifth digits of the hand. Dinosaurs, though, lacked the fourth and fifth digits.

"The implication is that it's virtually impossible to imagine how dinosaurs could have given rise to birds," says Feduccia.

Most paleontologists dismiss the impact of the new findings and argue that the evidence linking birds to dinosaurs is growing (SN: 8/23/97, p. 120).

Kevin Padian of the University of California, Berkeley criticizes the hand study. Burke and Feduccia's interpretation of bird hands rests on rules of development derived from modern animals, and Padian says that such assumptions cannot explain the pattern seen in dinosaurs. "The [hand study] neglects the fact that theropods clearly violate all the laws that they regard as inviolable. It's complete nonsense."

Paleontologist Lawrence M. Witmer of Ohio University in Athens calls the lung study intriguing, but he questions the conclusions. Theropod dinosaurs walked on two feet and had an immobile pubic bone, whereas crocodiles walk on four feet and have a pubic bone that shifts during movement. "The pelvic structures of crocodiles and dinosaurs were quite different. I'm not sure how to interpret the similarities that we see."

Witmer also says that the *Sinosauropteryx* specimen is squashed, making it difficult to interpret the tissue preserved in its chest cavity. —*R. Monastersky*