Circles in the Sky

Detecting the shape of the universe

By IVARS PETERSON

A spaceship and two stars as they would appear in a finite, multiply connected universe.

From "The Shape of Space" video © Geometry Center/Univ. of Minn.

The first of two articles examining alternative views of the shape of the cosmos.

Sailing high above Earth's surface, intrepid balloonists can travel in any direction and eventually end up where they started.

The possibility of circumnavigating the globe isn't obvious to a surface denizen who sees just an outstretched plain or ocean, however. Such a small patch of Earth looks flat, and one could hypothesize that the surface must either have an edge or extend indefinitely.

Moreover, even from a few thousand meters above the ground, it isn't readily apparent to an observer, convinced that Earth is a finite object, whether the slightly curved patch below represents a portion of the surface of a sphere, a doughnut, or an irregular blob.

Scientists face similar difficulties in discerning the overall shape, or topology, of the universe—in this case, the shape of three-dimensional space itself rather than that of a two-dimensional surface.

Mathematicians have discovered and investigated a wide variety of three-dimensional forms that could serve as models of three-dimensional physical space. Among the startling possibilities are those corresponding to a finite universe, in which a starship could blast off on a voyage of a few billion light-years in one direction and eventually return from another direction.

Astrophysicists and cosmologists are starting to pay attention to such bizarre notions. Instead of a limitless expanse of space studded with stars and dust, the universe could be finite and connected together in a complicated way.

Prompted in part by the enticing possibility of detecting the signature of the universe's topology in detailed maps of temperature fluctuations throughout space, a group of scientists and mathematicians met last October at Case Western Reserve University in Cleveland to compare notes.

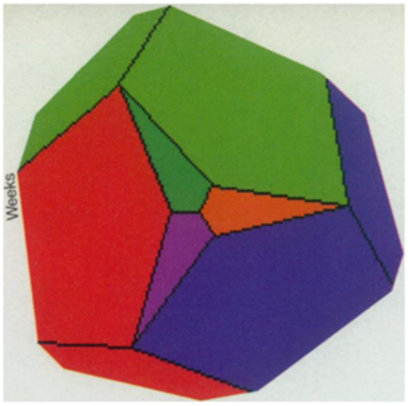
"There was a real sense of excitement at the meeting," says astrophysicist David N. Spergel of Princeton University. "We realized that, in this field, we had a lot to learn from mathematicians, and we had some interesting new questions for them. At the same time, it was exciting for mathematicians to see their results applied in this context."

According to current theory, the universe started out in an extremely hot, incredibly dense, highly contracted state. However, the Big Bang wasn't so much like a firecracker exploding into an existing space as the rapid expansion of space itself.

That expansion continued as matter cooled and condensed into dust, stars, and galaxies, drawn together by the force of gravity. As space kept expanding, those galaxies moved farther apart.

In the standard view, the ultimate fate of the universe depends on the density of matter within it. If the mass density were greater than a value called the critical density, gravity would be strong enough to reverse the expansion, eventually causing the universe to collapse into what could be called the Big Crunch.

In effect, such a universe would curve back on itself to form a closed space of finite volume. That space would have a positive curvature. A starship traveling in a straight line would eventually return to its point of origin.



Example of a polyhedron used to represent a negatively curved, or hyperbolic, three-dimensional manifold. The polyhedron's faces are color-coded to indicate which pairs are linked to create a multiply connected, finite space.

If the mass density were precisely equal to the critical value, the universe would go on expanding forever, though its rate of expansion would get closer and closer to zero. Its geometry would be called Euclidean, or flat, like the familiar geometry of lines and angles on a sheet of paper.

If the mass density were less than the critical value, the universe would also keep expanding forever, but at a constant rate. Such a space would be negatively curved and have a so-called hyperbolic geometry.

At present, observational data of various types suggest that the universe does not contain nearly enough matter to make it closed or even flat (SN: 1/3/98, p. 4). "There is growing evidence that we live in a negatively curved universe," Spergel says.

In contemplating a hyperbolic universe, cosmologists had generally assumed that such a space would have to be infinite rather than finite.

Mathematician William P. Thurston of the University of California, Davis and others, however, have demonstrated that three-dimensional hyperbolic space can take on a multitude of forms that are finite in extent. Any one of these forms could serve as a model of the universe's basic shape.

The underlying assumption is that physical space can be described in terms of a mathematical form known as a three-dimensional manifold. "There are lots of different possible manifolds that could represent space," notes freelance geometer Jeffrey R. Weeks of Canton, N.Y..

The surface of a ball is an example of a two-dimensional manifold. A small region of the surface looks nearly flat—like a piece of a two-dimensional plane. Earth's surface is a two-dimensional manifold because it looks essentially flat until one gets far enough away to see that it curves into a sphere. The surface of a doughnut, or torus, is also a two-dimensional manifold.

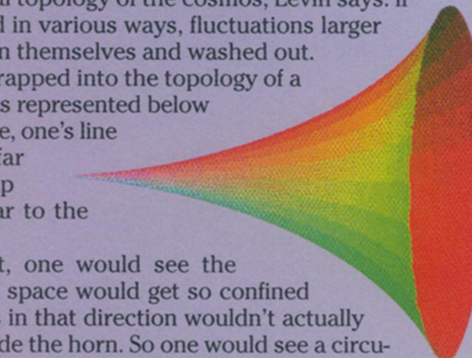
Inside the horn

Circles are not the only potentially detectable features in the cosmic microwave background that could point to the universe's fundamental shape.

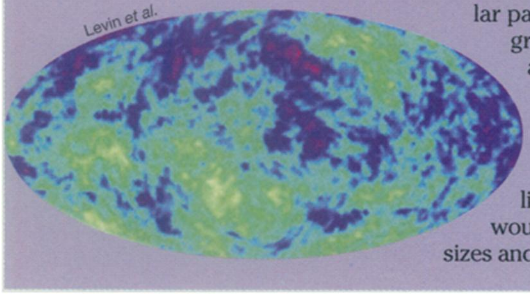
Janna J. Levin and Joseph Silk of the University of California, Berkeley, John D. Barrow of the University of Sussex in Brighton, England, and their collaborators have done computer simulations that illustrate what the pattern of hot and cold spots observed in the microwave sky might look like for a given topology.

The pattern of temperature fluctuations in the cosmic background radiation offers a critical test of the global topology of the cosmos, Levin says. If the universe is finite and connected in various ways, fluctuations larger than some scale would get folded on themselves and washed out.

For example, suppose space is wrapped into the topology of a hyperbolic manifold called a horn, as represented below in two dimensions. In such a universe, one's line of sight would extend infinitely far along the horn's axis but would wrap on itself in directions perpendicular to the axis (right).



Moving down the horn's throat, one would see the effects of topology, Levin says. The space would get so confined that a pattern of hot and cold spots in that direction wouldn't actually fit inside the horn. So one would see a circular patch in the microwave background where the temperature appears uniform (left).



It turns out that many three-dimensional hyperbolic manifolds have hornlike corners. "If you lived near one of these hornlike features," Levin says, "you would see flat spots, maybe in different sizes and orientations." —*I. Peterson*

Another way to think about the topology of a two-dimensional manifold is in terms of gluing together the sides of a rubbery rectangle. For example, a torus is simply a rectangle with opposite sides glued together. The first gluing creates a tube, and the second gluing connects the two ends of the tube to form a ring.

The same idea can be generalized to describe a three-dimensional manifold. For instance, one can try to imagine gluing together the opposite faces of a flexible cube to produce a hypertorus—the three-dimensional equivalent of a doughnut surface.

Thurston and Weeks were key figures in the development of a comprehensive catalog of closed three-dimensional manifolds, most of which appear to have a hyperbolic geometric structure. These weird shapes can be understood in terms of three-dimensional polyhedrons whose faces are glued together to create finite, multiply connected spaces.

The cosmological consequences are startling. If such a topology described the universe, what astronomers might think is a distant galaxy could actually be the Milky Way—seen at a much younger age because the light has taken billions of years to travel around the universe.

Can astronomers actually see all the way around the universe?

Obtaining evidence of a finite topology by looking for similarities between images of galaxies or quasars that appear in different directions, representing different times in their past, has proved extremely difficult. For one thing, quasars and galaxies evolve too quickly for astronomers to easily match up images of the same object in different epochs.

A clear signature may be written in the microwave sky, however. "What has invigorated the field is the realization that microwave background observations would be a clean probe of topology," Spergel says.

At its earliest moments, the universe is thought to have been a nearly uniform, scrunched plasma of photons and various particles, such as protons, electrons, and neutrinos. About 300,000 years after the Big Bang, that opaque plasma cooled enough to allow neutral atoms to form. Instead of being constantly scattered by particles, photons were then free to cruise the universe essentially unimpeded.

As viewed from Earth, which is immersed in this cosmological radiation, the photons appear to come from a sphere centered on the observer. Called the surface of last scattering, that sphere has a radius equal to the speed of light multiplied by the travel time since those photons' last interaction with matter. Because of the subsequent expansion of the universe, the photons have shifted to lower wavelengths, and they are now observable as the cosmic microwave background.

If the universe has a finite size, this expanding sphere "can wrap all the way around and intersect itself," Weeks says.

The intersection of one sphere with itself is seen as a circle. In a finite, multiply connected universe, an observer at the center of such a sphere would see the same circle of points in two different directions.

Spergel, Neil J. Cornish of the University of Cambridge in England, and Glenn D. Starkman of Case Western have proposed a scheme for detecting such circles in the sky.

A map of the cosmic microwave background shows tiny temperature fluctuations. The idea is to identify pairs of circles by matching point by point the pattern of temperature fluctuations along each circle. "That's what would tell you you're looking at the same circle of points in two different directions," Weeks says.

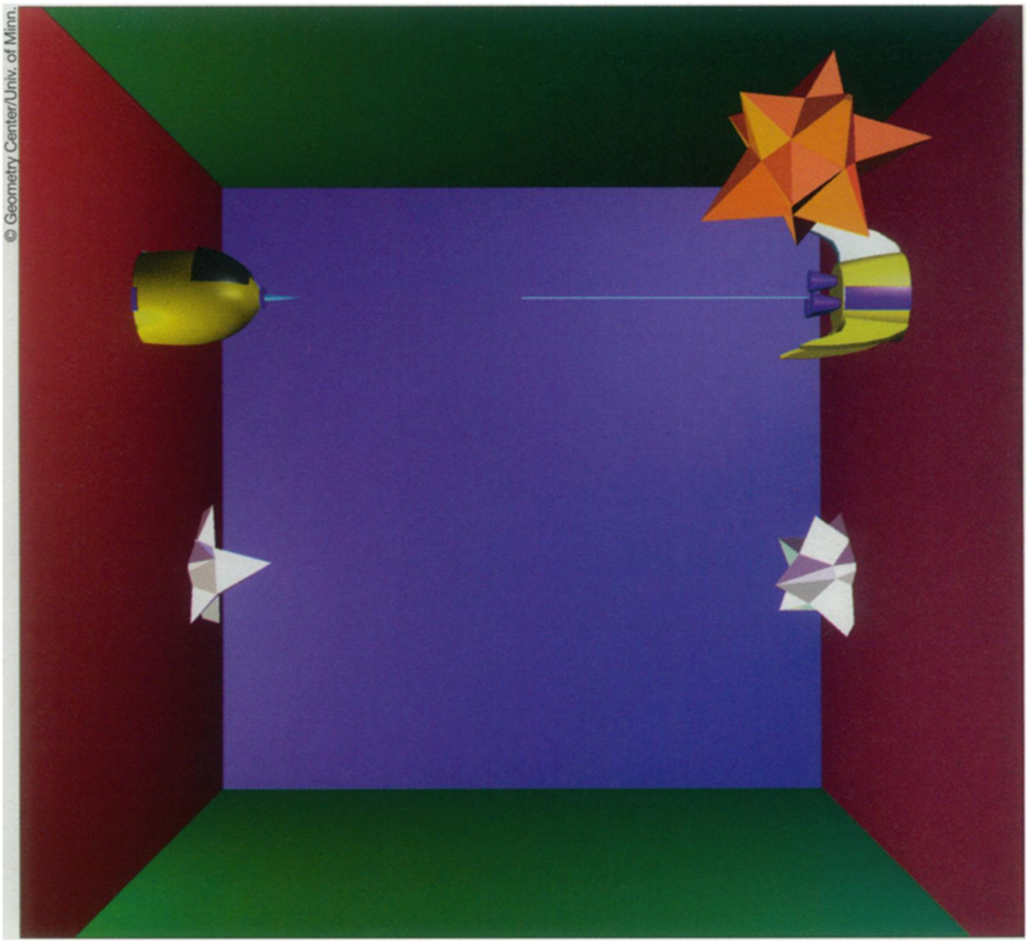
Moreover, "it's highly unlikely that the universe would be just the right size that you would get just one pair of intersections—that it's small enough that you can see around in only one direction," he adds. "There should be thousands of pairs of matched circles."

Once those circles have been identified, researchers can determine the specific type of three-dimensional manifold that fits the observations. "The pattern of circles is like a signature for the space," Weeks says. In the hyperbolic case, "we would probably get a small number of very large circles and a tremendously large number of small circles."

"If we find circles, we would actually be able to build a model of the universe," Starkman notes.

Microwave data from the Cosmic Background Explorer spacecraft (SN: 1/10/98, p. 20; 5/2/92, p. 292) had insufficient resolution to permit a fruitful search for circles. However, the launch in a few years of the sun-orbiting Microwave Anisotropy Probe (MAP) spacecraft and later of the European Space Agency's Planck satellite promise measurements of sufficiently high resolution to make a quest for circles feasible (SN: 6/7/97, p. 354).

"We don't know whether topology is important in cosmology, but it could be," Spergel says. "We'll be able to find that out reasonably soon, and if it is, it would be very exciting."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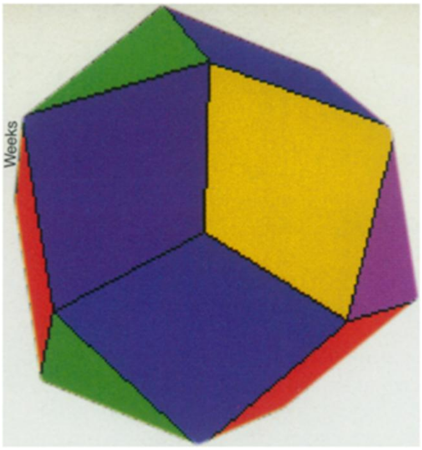
One way to picture what would happen in a finite, three-dimensional universe, represented in this case by a cube, is to imagine that the cube's faces were connected. Thus, an object apparently leaving the cube via one face would reappear as if it were emerging from another face.


Weeks

A polyhedron representational of a hyperbolic manifold.

The general theory of relativity posits that gravity is essentially a geometric effect—in other words, the theory links mass with the local curvature of space. Interestingly, it says nothing about the shape of the universe—the overall form of the three-dimensional spatial component of relativity's four-dimensional space-time.

Finding out this topology "would have a great impact on our vision of the universe," says Janna J. Levin of the Center for Particle Astrophysics at the University of California, Berkeley.

"It would tell us about physics beyond general relativity," Spergel adds.

Knowing the topology, researchers could independently determine the ratio of the universe's density to its critical value and reconstruct the state of the universe that gave rise to the cosmic microwave background. They could also gain insights into quantum gravity, quantum chaos, string theory, and other ideas at the forefront of efforts to construct a unified theory of force and matter.

"Cosmologists are used to thinking of looking out at the universe and measuring the prehistory of other regions of the universe," Cornish, Spergel, and Starkman conclude in the Jan. 6 Proceedings of the National Academy of Sciences. "If we are fortunate enough to live in a compact hyperbolic universe, we can look out and see our own beginnings."

"The mathematics of models for finite universes is rich," Spergel notes. "We've only just begun to explore the physics of those possibilities."

At the same time, many astrophysicists and cosmologists haven't given up on the notion that the universe is flat rather than negatively curved.

But that's another story. □

Next week: Saving the flat universe.