Cracking Kepler's sphere-packing problem

The familiar piles of neatly stacked oranges at a supermarket represent a practical solution to the problem of packing spheres as tightly as possible.

Now, a mathematician has proved that no other arrangement of identical spheres fills space more efficiently. That result—if verified—would finally solve a problem that has stymied mathematicians for more than 300 years.

Thomas C. Hales of the University of Michigan in Ann Arbor announced the feat this week and posted his set of proofs on the Internet (http://www.math.lsa.umich.edu/~hales/).

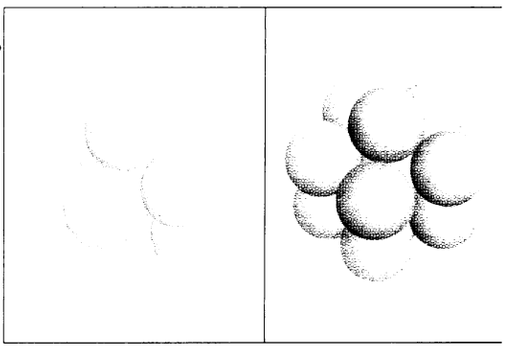
"These results are still preliminary in the sense that they have not been refereed and have not even been submitted for publication," he noted, "but the proofs are—to the best of my knowledge—correct and complete."

The proofs look convincing, says John H. Conway of Princeton University. "Hales has been careful to document everything, so that an auditor who has doubt over any particular point can actually go to the files and check that point."

When supermarket personnel stack oranges, the bottom layer consists of rows that are staggered by half an orange. Placing oranges in the hollows formed by three adjacent oranges in the first layer produces the second layer, and so on. Such an arrangement is known as face-centered cubic packing.

In 1611, Johannes Kepler asserted that this arrangement is the tightest possible way to pack identical spheres. In the 19th century, Carl Friedrich Gauss proved that face-centered cubic packing is the densest arrangement in which the centers of the spheres form a regular lattice. That left open the question of whether an irregular stacking of spheres might be still denser.

In 1953, László Fejes Tóth reduced the Kepler conjecture to an enormous calculation involving specific cases and later suggested that computers might be helpful for solving the problem. Hales recently worked out a five-step strategy to implement that approach.



Ferguson

Part of the proof of the Kepler conjecture shows that clusters of identical spheres arranged with face-centered cubic packing (left), like neat stacks of oranges in a grocery store, fill space more efficiently than the pentahedral prism (right).

In a key step, Michigan graduate student Samuel L.P. Ferguson proved that an irregular arrangement based on a structure known as a pentahedral prism is less dense than the face-centered cubic packing. That was a crucial finding because preliminary computer experiments had indicated that the pentahedral prism might be a counterexample to Kepler's conjecture.

"There were a number of tricky things about solving the problem," Ferguson says. Techniques developed to handle this case were useful in carrying out the other steps, he explains.

The complete, five-part proof appears in a series of articles totaling more than 250 pages. The computer programs and data files take up 3 gigabytes of memory.

Hales used a variety of computational techniques to ensure the accuracy of his calculations. He also worried about the possibility of errors introduced by defective computer chips and any faults in the way a computer translates a program into instructions to a microprocessor.

"There is certainly quite a lot of room for error," Ferguson says. As one check, he and Hales independently wrote computer programs to verify important steps.

Nonetheless, "the problem with such proofs is . . . their length, not the involvement with the computer," Conway says. "A long proof is inevitably weaker than a short one just because there are so many more places where a slip might have been made."

If it holds up, the Hales proof demonstrates that Kepler was right. This feat may not, however, represent the last word on the problem.

"I don't see why there shouldn't be a very short proof involving totally different ideas," Conway says, "and [I] would hazard the guess that there is." *—I. Peterson*

Laser beam can pop out single cells

A laser that cuts out a cell and flips it into a waiting test tube offers researchers a way to remove single cells from a tissue sample in less than 30 seconds.

The technique offers a quick means to choose cells for DNA screening tests while minimizing contamination. This is especially important when applying the polymerase chain reaction, or PCR, a widely used method for making many copies of DNA or RNA (SN: 10/23/93, p. 262). Karin Schütze and Georgia Lahr of the Academic Hospital in Munich describe their new technique in the August NATURE BIOTECHNOLOGY.

First, the researchers affix a slice of tissue to a thin plastic sheet and mount it on a glass slide above a highly focused ultraviolet laser. Looking through a microscope, they identify a particular cell, then burn around its edges with the laser, cutting out the cell like a cookie from a sheet of dough.

Then, by doubling the power of the laser but adjusting it to focus below the target cell, they launch the individual cell vertically off the glass slide and catch it with the cap of a test tube. The cell rides the stream of photons from the laser "like a surfer on top of a wave," says Lahr. The cell "can be beamed several millimeters away, even against gravity," she reports.

To demonstrate the selectivity of their technique, called laser pressure catapulting, Schütze and Lahr collected individual colon cancer cells in test tubes. An analysis of the DNA of a single cell revealed the presence of a mutation that has been linked to colon cancer.

One advantage of the method is that the person preparing the sample never has to touch the cell. Earlier methods used fine needles to dissect cells from a tissue sample—a slow, tedious process. A disadvantage of laser pressure catapulting is that it kills the cell, although the genetic material remains intact.

Other researchers have developed similar laser-based techniques to isolate clusters of cells. In one such method, a team at the National Institutes of Health in Bethesda, Md., uses an infrared laser to heat a plastic film laid on top of a slice of tissue. The film melts above the target cells and sticks them to a plastic cap held above the sample. When the cap is lifted away, it carries the cells with it. This method also kills the cells.

Yet another variation, called anchored cell analysis and sorting, works on live cells growing on a piece of heat-absorbing plastic film in a petri dish. A laser beam is shone onto the film around the cells of interest, fusing a microscopic island of plastic to the dish. When the rest of the film is peeled off, it takes the surplus cells with it, leaving the targeted cells behind. This technique was described in 1985 by Melvin Schindler of Michigan State University in East Lansing.

The live cell then "can be examined, manipulated, and potentially cloned," writes Schindler in a commentary accompanying the German study. All these techniques should ease the preparation of DNA libraries that catalog the patterns of gene expression in cells, he adds. *—C. Wu*