Computing at the Edge

Capturing a flame's flicker, an ink jet's splatter, and other shifting shapes

By IVARS PETERSON

Researchers have used a method called fast marching to calculate the shortest paths (geodesics) across various surfaces, including this computer-generated representation of a human head.

The foam-flecked fringe of a breaking ocean wave marks a rapidly changing, intricately woven boundary between water and air.

Every detail of that turbulent behavior results from complex responses to physical forces acting at the interface between two different materials. Similar boundary effects occur amid a fireplace's wavering flames, in milk swirled into a cup of tea, at the freezing edge of a snowflake, and among ink droplets ejected from a microscopic nozzle.

To study and predict boundary effects, researchers have struggled to accurately capture erratic, complicated behavior in a computer model. In recent years, however, two novel schemes for calculating what happens at rapidly evolving interfaces have provided new insights into a variety of physical processes important in technologies such as semiconductor manufacturing and ink-jet printing.

"By suitably writing the equations for a propagating interface, one can find a way of borrowing numerical techniques from computational fluid mechanics to solve these equations of motion," says mathematician James A. Sethian of the University of California, Berkeley.

Adds Stanley Osher of the University of California, Los Angeles, "You can compute things that you couldn't compute before."

Researchers have now adapted the same techniques to detect edges in scanned medical images, find optimal paths for robots negotiating obstacle-strewn courses, and create special effects for movies.

Consider an ice cube floating in a glass of water. The boundary between ice and water shrinks as the ice melts, and it grows as more water freezes.

Each point along such an interface has a speed, which depends on a variety of physical factors. For a floating ice cube, the speed at which a given point moves in or out might be determined by the local temperature difference between the ice and the water.
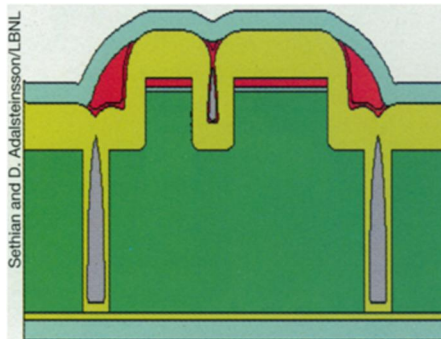
To model a moving interface, researchers conventionally have represented the boundary as a set of connected points—like a system of buoys loosely linked together with pieces of rope. The idea is to move each marker with the appropriate speed, according to the relevant equations of motion, and track the resulting changes in the interface's shape.

Such computational methods, however, often fail when the interface shape fluctuates rapidly, develops sharp corners or cusps, breaks up into smaller pieces, or merges with other features. In

Recent simulations based on level-set methods (left) compare favorably with experimental studies (right) of the various etching and deposition steps required to create a complex integrated circuit chip profile.
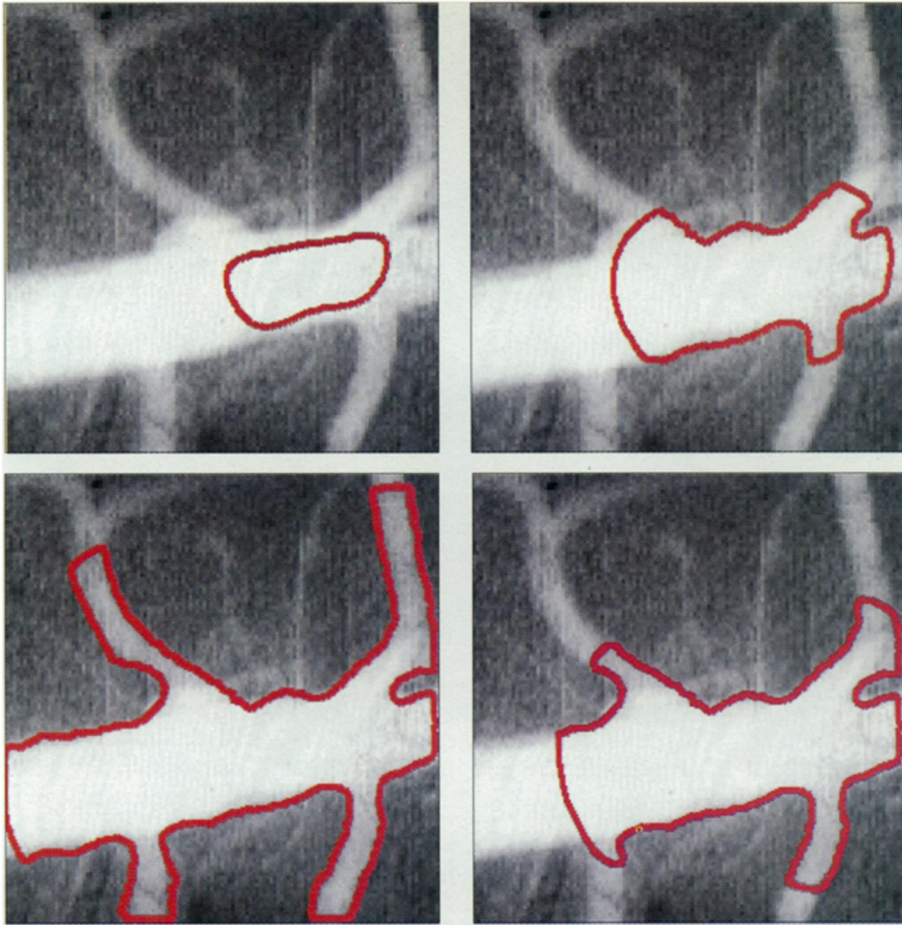
effect, the ropes connecting the buoys get tangled or severed.

About a decade ago, Sethian and Osher discovered an ingenious way to skirt such problems. They called their method the level-set approach. The mathematical term "level set" refers to a contour, much like the curve linking points of equal elevation on a topographic map.

Suppose that the two-dimensional interface between materials starts out as a circle. Then, think of the circle as a slice through a three-dimensional surface—a cone, for example. This surface is known as a level-set function. If the chosen surface were a topographic map, the interface would be defined as the sea-level (or zero-elevation) contour.

Researchers have tremendous freedom in choosing an appropriate three-dimensional surface for a given interface. Above and below the zero-elevation contour, the smooth shape could bulge out or narrow to a point, for example. The trick is to find a surface that changes smoothly over time even if the contour by itself does not.

Next, the mathematicians consider the physics governing the contour's movement, such as the inward march of a melting ice cube's edge. They then calculate how points on the three-dimensional surface would move in accordance with differential equations describing that physics. To find the shape and location of the two-dimensional boundary at any moment, they simply determine the evolving surface's zero level at that time.

Although the level-set approach trades the problem of a moving curve for one involving a moving surface, it offers significant computational advantages, Sethian

Example of how the edge-detecting capability of the level-set method, which starts with an expanding circle, outlines an artery in a medical image (clockwise from top left).

Berkeley (Calif.) National Laboratory and his collaborators are now working on what they term "computer-aided cytology." Their goal is to develop a reliable, automated method for analyzing microscope images of cells, Malladi says.
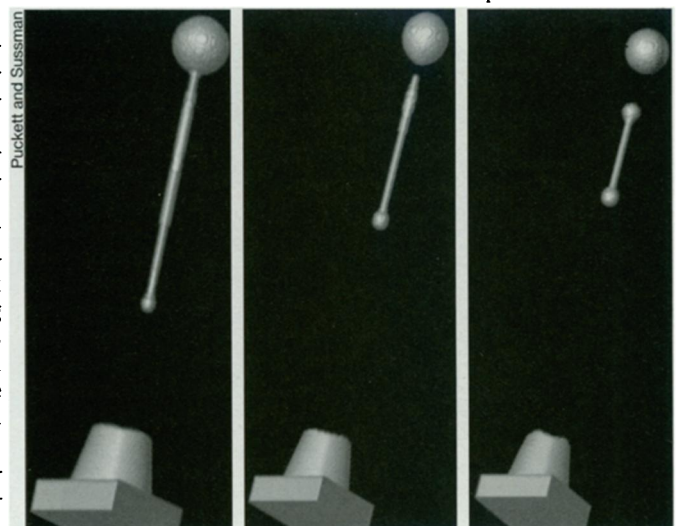
Working independently, Osher and his colleagues have applied the level-set approach to model the growth of crystals and thin films on surfaces. Because the thin-film model describes dozens of constantly moving and merging islands of aggregating atoms, the level-set method is essential for practical simulations of the growth process, Osher says.

Researchers have also simulated processes in which not just two but three phases, or components, interact. One example models the spread of oil through water beneath ice.

At the same time, simulations of two-phase flows show promise as convincing computer-generated images of splashing water and other complicated subjects on movie screens. "Flames can also be done this way," Osher notes.

One intriguing set of applications involves image processing and computer vision. Video enhancement techniques developed by Osher and Leonid I. Rudin of Cognitech in Pasadena, Calif., smooth out or remove static and other distortions while keeping edges sharp and clear. That approach is based on the notion that the edges of objects obscured by electronic noise can be described by the equations used to model shock waves traveling through fluids.

Such image restoration has already played a critical role in the courtroom. During the 1992 Los Angeles riots, video cameras in helicopters captured several men pulling a truck driver out of his vehicle and severely beating him. Defense lawyers argued that the videotape was too low in quality to link one particular person to the attack. Rudin's analysis, however, revealed a distinctive dark patch in the same

says. Whereas a two-dimensional front can split up and become wildly distorted, the corresponding three-dimensional surface has been chosen so that it undulates and twists in a well-defined, mathematically tractable manner.

The strategy also works for tracking a three-dimensional interface. Though much harder (if not impossible) to visualize, it considers the interface as a closed surface, like a ball or distorted blob, and follows the evolution of its corresponding level-set function in four-dimensional space.

"Armed with the level-set perspective from this higher-dimensional space, we can efficiently compute solutions to a wide collection of problems," Sethian says.

Sethian and his collaborators have explored many varied applications of the level-set method. They have modeled a bubble of one liquid rising within a liquid of higher density and also have simulated the etching of a semiconductor surface during manufacture of an integrated-circuit chip.

The investigators have recently applied their scheme to the automatic extraction of anatomical features from medical images, such as magnetic resonance scans. In the past, physicians typically drew outlines by hand to identify spots requiring further examination or to measure the

size of enclosed regions to see how they change from one scan to another.

To outline an organ or some other feature automatically, a computerized front starts as a small circle inside the area of interest in a digitized image. That initial spot is then allowed to spread out at a speed that depends on the change in brightness from one pixel to the next. When a portion of the front hits an abrupt brightness change, presumably marking an edge, it slows dramatically. Eventually, the front outlines the required feature.

One medical imaging company recently adopted this approach to locate and measure fatty deposits and track heart-wall motion in images obtained using a new electron-beam scanner. "These very fast scanners are using the latest algorithms to help identify and measure very delicate features," Sethian says.

Also taking advantage of level-set methods, Ravikanth Malladi of the Lawrence

Simulation of a tailed droplet formed by a jetting device.

location as a tattoo on the defendant's arm. The defendant was convicted.

Plastic surgeons have recently expressed interest in the various image-processing and analysis techniques developed by Osher and his colleagues. They would like to be able to classify complicated shapes, such as bone structures and facial features, by reducing the data obtained from a scan to a small number of numerical parameters that characterize the features of interest.

Sethian has developed an efficient technique for rapidly calculating how curved interfaces move in the particular case when the front always advances in one direction and never backtracks. He calls it the fast-marching method.

For a two-dimensional interface, he sets up a surface with the property that if he slices it at a certain height, he immediately gets the position (and shape) of the front at the corresponding time. In effect, the physics is built into the shape to encode the interface's behavior over time.

That approach lends itself to the problem of determining the shortest path that a robot can take through a maze of narrow corridors to reach its destination—or that shoppers can use to find their car in a crowded parking lot.

Imagine a front—like a ripple—expanding outward from a starting point. Depending on what obstacles or surfaces different parts of the front encounter, the front expands faster in certain directions than in others, sometimes splitting into two or more fingers, sometimes merging again.

For example, a parcel-laden shopper coming out of a store would be the ripple's starting point. Other people's cars, trash cans, and piles of snow might slow or split the front. Eventually, some point on the front would reach the shopper's car. The path that had been followed by that point would represent the shortest route to the car.

Sethian and Ron Kimmel, now at the Technion Israel Institute of Technology in Haifa, Israel, described in the July 21, 1998 PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES how to use this method to compute the shortest paths, called geodesics, from point to point across hilly terrain and other computer-generated landscapes.

The level-set approach by itself, however, doesn't solve all tricky computational problems involving moving interfaces, says Elbridge Gerry Puckett of
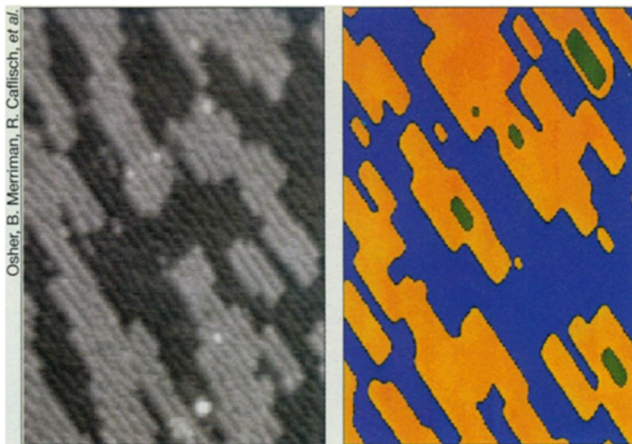
A scanning tunneling microscope image (left) of a patchy indium arsenide film (light areas) shows roughly the same pattern as a computer simulation (right) based on the so-called island dynamics model of crystal growth.

the University of California, Davis.

A water drop breaking up into smaller droplets as it moves through still air, for example, would always maintain the same total volume. Level-set methods don't automatically include such a constraint.

Puckett is interested, for example, in modeling the formation of ink droplets ejected by the microscopic nozzles of ink-jet printers. Current technology allows the generation of ink drops with diameters of 25 to 100 micrometers at rates of up to 4,000 drops per second.

The trouble is that, under most operating conditions, the jet rarely consists of a spherical drop. Created by a pressure pulse, a droplet typically exits the nozzle with a tail. This tail pinches off and breaks up into a few satellite droplets that are generally smaller and slower than the main droplet. Because these satellite droplets follow different paths, they miss the target, and so they slightly blur the final printed image.

Researchers have had difficulty simulating the details of the droplet-formation process to learn how to get rid of satellite droplets. Standard methods produce inaccurate results, Puckett says.

One promising solution, based largely on research by Mark Sussman of the University of California, Davis, couples the level-set approach with the conventional volume-of-fluid method to create a powerful hybrid technique for modeling the entire jetting process.

Although the research results aren't published yet, several companies have already expressed interest in the simulation technique. The new method "won't solve everything, but if you know what you're doing, you can do a lot," Puckett says. "You need to tailor your method carefully to the problem at hand."

The level-set method has proved to be phenomenally successful as both a theoretical and a computational device, Osher remarks. "The possibilities are endless." □