

MATHEMATICS

Quaternions

"A Classic of Science"

Sir William Hamilton's Synthesis of Three Dimensions of Space With One of Time into a Four Dimensional World

ELEMENTARY SKETCH OF THE NATURE OF THAT CONCEPTION OF MATHEMATICAL QUATERNIONS, WHICH IS DEVELOPED MORE IN DETAIL BY SIR W. R. HAMILTON, IN HIS RECENTLY PUBLISHED VOLUME OF LECTURES ON THAT SUBJECT. In Appendix to the life of Sir William Rowan Hamilton, by Robert Perceval Graves. 3 volumes. Dublin: Hodges, Figgis & Co.; London: Longmans, Green & Co.; 1889.

THE WORD "Quaternion" requires no explanation, since, although not now very commonly used, it occurs in the Scriptures and in Milton. Peter was delivered to "four quaternions of soldiers" to keep him; Adam, in his morning hymn, invokes air and the elements, "which in quaternion run." The word (like the Latin "quaternio," from which it is derived) means simply a *set of four*, whether those "four" be persons or things.

(2) But the question arises, what special connexion has the *number Four* with mathematics generally, or with that branch of mathematical science in particular, to which the "Lectures on Quaternions" relate?

(3) One general form of answer to this question is the following:—that in the mathematical quaternion is involved a peculiar synthesis, or combination, of the conceptions of *space* and *time*; and that while *TIME* is usually pictured or represented by metaphysicians under the figure of a *line*—a single stream with its *ONE* current—an unique axis of progression, *SPACE* is, on the contrary, imagined or conceived in connection with *THREE* distinct axes, three lines at right angles to each other;

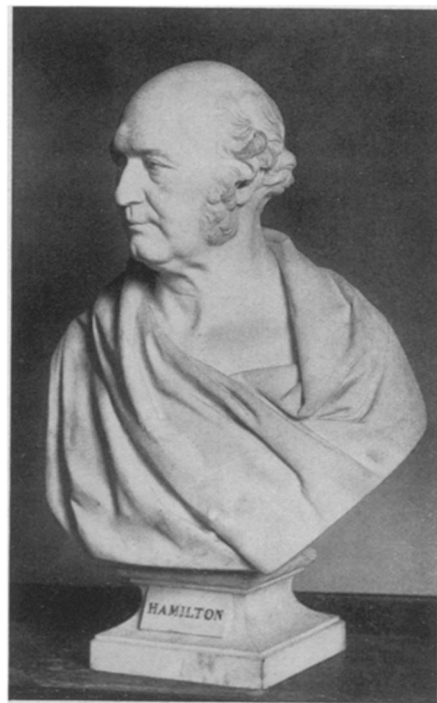
This popular account of the principles on which his Quaternion Calculus was founded was written by Sir William Hamilton shortly after the publication of his "Lectures on Quaternions." "It is, therefore," says his biographer, "to be accepted as a fully-matured exposition of its subject." The Calculus of Quaternions is one of the foundations of modern mathematico-physical theories of the universe.

such as the three edges of a room, which meet at a corner of the ceiling, or of the floor, one vertical and two horizontal; height, length, and breadth. In *time*, we have only the forward and the backward, looking before and after. In *space*, there is not merely the contrast between the directions of upward and downward, but also between those of southward and northward, and again between westward and eastward. Time is said to have only *one dimension*, and space to have *three dimensions*. The former is on *unidimensional*, the latter a *tridimensional progression*. The mathematical *quaternion* partakes of *both* these elements; in technical language it may be said to be "time plus space," or "space plus time": and in this sense it has, or at least it involves a reference to, *four dimensions*. In an unpublished sonnet to Sir John Herschel, entitled "The Tetractys" (a Greek work equivalent to the Latin Quaternio), the author of the Lectures introduced the two following lines, which give, in the shortest possible form, an expression of the view which has been in the foregoing remarks unfolded more at length:—

"And how the One of Time, of Space the Three,

Might in the Chain of Symbol girdled be."

(4) Those who are entirely unacquainted with mathematical science may yet derive, from what has been above remarked, a sufficient preliminary insight into the nature of the speculations and inquiries to which the "Lectures on Quaternions" relate. A philosophical, if not a technically scientific, knowledge of the author's general aim, and of the idea which has guided him, may in this way be easily attained. But a very moderate acquaintance with the conceptions of geometry will suffice to render intelligible, from another point of view, the importance which the author attaches to the number Four in mathematics.



SIR WILLIAM ROWAN HAMILTON
—was born in Dublin, Ireland, in 1805 and died in the same city in 1865. He was an astronomer and poet in addition to being one of the great mathematicians of the world.

(5) As early as the first book of Euclid's *Elements*, an attentive student is (or may be) led to consider the *relative length*, and also the *relative direction*, of one straight line as compared with another. Thus when Euclid shows, in his very first proposition, how to construct on a given base AB an equilateral triangle ABC, he virtually teaches how, when one line AB is proposed or *given*, to draw a *new line* BC (or AC), which shall in length be *equal* to the given one, and in *direction* shall make with it an *angle of sixty degrees*, namely, the angle ABC (or BAC), which is the third part of 180 degrees, or of two right angles.

(6) In this elementary example, if the length of the given *base* AB be taken as the standard of length, and be on that account called *unity*, or *one*, then the length of the *side* BC (or AC) of the triangle must also be denoted by the same number, *ONE*; and these *TWO*

NUMBERS, *one* and *sixty*, serve in this view to define, or to describe, the *length and direction* of the new or constructed line BC; at least if the latter number (*sixty*) be combined with the consideration of a certain *hand*, or *direction of rotation*, towards which the old line BA may be conceived to *turn*, in the plane of the triangle (or of the paper), as indicated by the *curved arrow* in the figure.

(7) The foregoing view, although not precisely the same with that adopted by Euclid himself, in his exposition of the elements of geometry, is at least consistent therewith; and has been made the basis of an important and modern method of calculation, respecting *directed lines in one plane*, which seems to have been first introduced about the commencement of the present century, by Argand in France, and for which Professor De Morgan of London has lately proposed the name of *Double Algebra* because it recognizes and employs *two numerical elements* (such as the numbers 1 and 60 in the foregoing example), as required for the joint determination of the *length and direction of a straight line*. And it is now to be shown what is the nature of the passage that has been made, by the author of the Lectures on Quaternions, from such a *double system* of algebraic geometry, to what may be called, by analogy and contrast, a *quadruple system* of calculations respecting directed lines, or a system of QUADRUPLE ALGEBRA.

(8) This passage from the one system to the other may be said to consist mainly in the consideration of the *variable plane of an angle*. If, after tracing the equilateral triangle ABC on a *card*, which at first rests on a horizontal *table*, we then lift up that card, with the figure traced thereon, and lay it on a sloping *desk*, the triangle in its new position takes also a *new aspect*; it faces a different *region* of space, and may be conceived to *look at*, or be looked at by, a *new point of the heavens*, which is *not now* the *vertical point* (or zenith), as before. This *new aspect of the figure*,

or of the *plane* (or desk) on which it is now situated, is the *new circumstance* introduced, in the transition from Double to Quadruple Algebra. And in fact it is easy to see that this new circumstance, of the *varied position of the figure*, namely, of the triangle, or simply (if we choose) of the ANGLE ABC, requires the consideration of *two new numerical elements*. For we have now *two new questions* to answer, or *two new things to determine*: namely, 1st, the *slope of the desk* (or inclination of the plane), suppose forty-five degrees, conducting to a *first new number*, 45; and 2nd, the *direction of the edge* (or, technically speaking, the line of the nodes), where that slope meets the table, and which may deviate from the line of north and south by any other number of degrees, suppose seventy, giving thus a *second new number*, in this case 70.

Science News Letter, March 4, 1933

The dark film which appears inside an aluminum pan when certain kinds of water are boiled in it is reported to have no effect on health, and can be easily removed by stewing apples or other acid fruits in the pan.

PHYSIOLOGY

Growth-Checking Substance May Control Cancer Tissue

GROWTH of tissues, stimulated by the presence of naturally occurring compounds containing the sulfur-hydrogen combination known as "sulfhydryl" and designated with the chemical symbol SH, is checked by compounds containing the same combination in a partially oxidized condition. This, in summary, is the result of research by Dr. Frederick S. Hammett of the Lankenau Hospital, Philadelphia. If sustained by further experiments, Dr. Hammett's discovery will be of immense importance both scientifically and practically.

Dr. Hammett announced his discovery that sulfhydryl-containing compounds accelerate growth at a meeting of the American Philosophical Society three years ago. At that time he also suggested that a growth-checking action might be expected of the same compounds in an oxidized or partially oxidized condition.

Following up this lead, one of his

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cent. By 1950 it is estimated that coal would furnish only 46 per cent. of the country's power, while 45 per cent. would come from oil and gas, and 8 per cent. from water power.

Other findings of the study by Prof. Hutchinson and Mr. Breitenstein are:

In 1930, the energy supply per capita, expressed in millions of British Thermal Units, was 73, while in 1950 it is expected to be 94.

Whereas bituminous and anthracite coal accounted for 60.3 per cent. of the total energy derived in 1930, it will account for only 46.6 per cent. in 1950.

Petroleum and its natural products, including also natural gas and natural gas gasoline, will show a marked rise. Accounting for only 31.6 per cent. of the total energy derived in 1930, they will account for 45.3 per cent. in 1950.

Water power will account for exactly the same percentage of the total energy derived in 1950 as in 1930, namely, 8.1, although the energy applied by hydro-power will be greater than in 1930.

Science News Letter, March 4, 1933

JOSEPH PRIESTLY

—born 200 years ago, an early American scientist (by immigration) explains why he held to the phlogiston theory

IN THE NEXT CLASSIC OF SCIENCE