

## MATHEMATICS

# Fold Paper to Learn Geometry

A recreational activity to keep a child's nimble fingers busy when adapted to geometrical shapes becomes an intriguing teaching aid for the mathematics class.

By RUBY YOSHIOKA

► PAPER FOLDING, an ancient recreational art, has come to the aid of mathematicians.

Paper folding as a universal pastime for children in Japan has been developed through many centuries. It is known as "origami," "ori" from the word fold and "gami" from "kami" meaning paper. Many intricate designs and shapes have been constructed.

Widely popular in the United States today among both old and young, origami is helpful in fields other than mathematics. Specialists in occupational therapy use paper folding in certain cases to help children regain muscular control and coordination in their hands and fingers.

For the bedridden and the aged it has therapeutic value, helping these patients dispel long hours of boredom.

Paper folding is an intriguing and enjoyable way of illustrating geometric principles and mathematical relationships. Shaping angles and lines with paper appeals immediately to virtually everyone.

## Paper Folding as Visual Aid

Visual aids are being used more and more in the teaching of mathematics. These aids are especially appropriate since the study of geometric shapes and concepts is being introduced earlier in many schools in recent years, some of them starting even in the fifth and sixth grades.

For many people thinking in the abstract and visualizing objects from lines on a printed page is very difficult. Paper folding will help these persons understand spatial relationships and visualize geometric shapes more clearly and vividly. By folding and handling the paper, the square or triangle becomes a concrete object and not mere lines on a static printed page.

Some persons have more difficulty than others in seeing geometric forms in the objects around them. Taught correctly, children will see that they live in a world of geometry. Boxes, cans, trees, snowflakes, crystals of salt or a chair are all geometric shapes. Students will become more aware of the curves, lines and angles in architectural designs, such as domes on buildings, bridges and gateways. Removed from the printed page, triangles, squares, curves and lines take on life and interest.

Mathematics teachers can take advantage of a child's innate desire to create and do things with his hands, and introduce paper folding in the classroom.

Here are a number of simple mathematical paper folding problems that combine both fun and instruction. All one

needs is paper, pencil, a ruler and scissors. Paper that creases easily is preferable. Heavy wax paper is excellent for this purpose.

## To Find Area of Triangle

Take a sheet of paper and mark it with three widely spaced points, indicating them as A, B and C. Make a crease from A to B, B to C and C to A. Cut along the creases to form a triangle, the simplest figure having straight sides that can completely enclose a plane.

To find the area of this triangle, make a fold from the vertex of the triangle perpendicular to the base. This line is the altitude of the triangle. Mark the point of intersection of the altitude with the base D. Bring vertex B down to meet point D. Bring vertices A and C also to point D (see Figs. 1a and 1b). You have now formed two rectangles of equal size. By

finding the area of the rectangles, the product of the length times the width, and taking twice this value, the area of the triangle is obtained.

Try this with differently shaped triangles.

Can you show by this means that the sum of the angles of a triangle is 180 degrees?

## Construct an Equilateral Triangle

Construct an equilateral triangle by measuring off five inches for one side AB. Find the midpoint of this base and fold the sheet perpendicular to the base. Measure a line five inches from point A to the perpendicular fold. Mark this point C. Make a fold from A to C, which should also be five inches in length, and from B to C. Cut along these creases for your equilateral triangle.

All the angles of an equilateral triangle are equal. Since the sum of the angles of a triangle is 180 degrees each of the angles should be 60 degrees. This can be checked with a protractor if you have one, or by placing one angle over the other to see if they are equal.

Fold the equilateral triangle in half and you will have two right triangles with

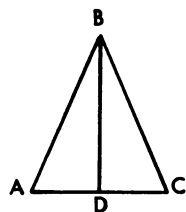


Fig. 1a

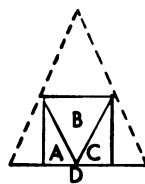


Fig. 1b

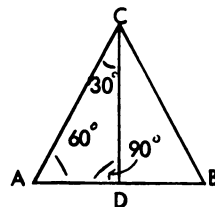


Fig. 2

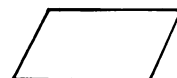


Fig. 3a



Fig. 3b

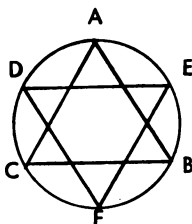


Fig. 4

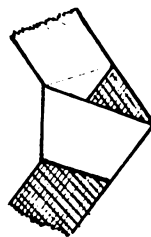


Fig. 5

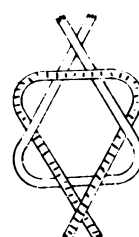


Fig. 6a

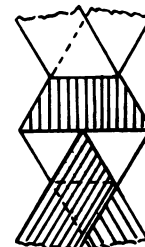


Fig. 6b

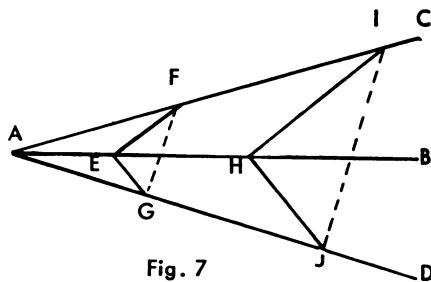


Fig. 7

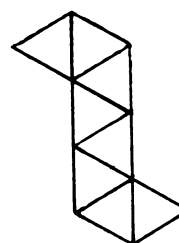


Fig. 8

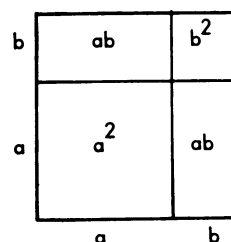


Fig. 9

**GEOMETRY IS FUN**—Diagrams of a number of paper folding problems that illustrate some geometric principles and relationships.

angles 90, 60 and 30 degrees. The sum of the angles is 180 degrees in each case. Mark the midpoint of the base of the triangle D. In a triangle with angles measuring 90, 60 and 30 degrees, the longer side is always twice as long as the shortest side. Show this by folding. See Fig. 2.

### Area of a Parallelogram

A parallelogram is a quadrilateral whose opposite sides are parallel. Cut out a parallelogram as shown in Fig. 3a. To show that the area of this figure is the base times the altitude, fold the end of the parallelogram to form a right triangle as shown in Fig. 3b. Cut along the fold and fit onto the other end to form a rectangle.

Any triangle is always of equal area to a parallelogram having a base equal to that of the triangle. Cut out a triangle and then a parallelogram with a base equal to that of the triangle and an altitude half that of the triangle. Fold the triangle as you did before to find the area and fold and cut the parallelogram to find its area. Do the areas seem to be the same?

### Make a Six-Pointed Star

Can you make a six-pointed star from a circle? See Fig. 4.

Cut out a circle and fold at a diameter and crease. Open it up again and fold another diameter. Where the two folds intersect is the center of the circle. Now fold a chord AB so that its arc touches the center of the circle. Crease. Fold another chord BC to the center as before, and a third CA. Fold the vertex of each of the angles thus formed to the center of the circle and you will have three new angles. The vertices of these six angles form the points of the star.

From the star, fold a hexagon or a six-sided figure.

### Knots Form Geometric Shapes

Take a fairly long half-inch strip of paper and tie a simple knot pressing the folds down carefully and you have formed a pentagon as shown in Fig. 5.

Take two one-half inch strips of paper of different colors and intertwine them as in Figs. 6a and 6b. Secure the knot carefully and flatten, forming a hexagon. More complex polygons can be formed by more complicated knots. How many persons think of knots in terms of geometric shapes? This subject should be a challenge to many.

### Parallel Sides of a Triangle

If two triangles have two sides of one parallel to the corresponding two sides of the other triangle and straight lines can be drawn from an external point through their vertices, then the third sides of the triangles are parallel to each other.

Take a sheet of paper and fold along the heavy lines as indicated in Fig. 7, being sure that lines FE and IH are parallel and EG and HJ are parallel. Now fold along the dotted lines FG and IJ. These lines should also be parallel.

Measure the angles of the two triangles with your protractor. What significant rela-

tionship do you find in the angles of triangles whose corresponding sides are parallel?

### Construct an Octahedron

Three-dimensional forms of solid geometry also can be constructed by paper folding. A simple structure is shown in Fig. 8. By cutting out and folding along the lines as shown in the diagram, an eight-sided structure or octahedron is formed.

Paper folding is not the only method of constructing three-dimensional geometric shapes. Making geometric models with cardboard, glue and other equipment is an elaborate art that has been highly developed. Beautiful intricate shapes in different colors have been created.

Other means of constructing three-dimensional shapes have been devised, such as toothpicks or similar sticks and preformed joints or modeling clay. Basic geometric solids including cones and cylinders are on the market to help the student. Models built by the students themselves, however, seem to be the most helpful.

### PUBLIC SAFETY

## Food Stockpiling Urged

**Following an atomic attack, farm production will drop to about one-fourth of normal and it is imperative to stockpile a sufficient food reserve, says a California scientist.**

► MASSIVE stockpiling of fuel and food for use after an atomic war is advocated by Dr. Albert Bellamy, professor of biophysics, emeritus, at the University of California, Los Angeles.

Dr. Bellamy is a former chief of the radiological section, California Office of Civil Defense, and is a consultant to the civil defense liaison branch of the Atomic Energy Commission's division of biology and medicine.

He points out that non-military defense systems against nuclear attack should be considered in two broad phases:

1. Casualty prevention during the acute phase of nuclear attack.

2. Acceleration of national recovery during a much longer post-attack period.

The first phase, perhaps a two-week period concerned with blast-resistant fallout shelters and emergency supplies, has been widely discussed. But the second phase, which may extend over a period of several years, has received little attention.

"Surviving populations in and near our great cities and industrial centers are likely to be greeted with scenes of destruction not unlike those of Hiroshima and Nagasaki," Dr. Bellamy says.

"On the other hand, people in rural areas and smaller communities are likely to emerge from fallout shelters into familiar and undamaged surroundings.

"But because today's agriculture depends to a large extent on machines—such as tractors and trucks—agriculture will be at a

### Fold an Algebraic Equation

Even algebraic expressions can be demonstrated by paper folding.

Take the equation  $(a + b)^2 = a^2 + 2ab + b^2$ . Cut a square piece of paper. Fold down one side parallel to the edge. Fold down an equal distance in the other direction as shown in Fig. 9.

Mark the sections as shown in the diagram. Cut out each section and arrange the pieces according to the equation and the relationship becomes immediately evident.

Much learning by children is through doing and feeling. A creative teacher can devise paper folding problems of his own to explain geometric principles where concrete illustrations will help.

For those interested in the necessary materials for mathematical paper folding and a series of experiments that can be worked out at home or in the classroom, unit No. 267 of THINGS of science is available at 75¢ each, from Science Service, 1719 N St., N.W., Washington 6, D. C.

• Science News Letter, 83:138 March 2, 1963

virtual standstill because of lack of gasoline and diesel supplies. Petroleum fuels, therefore, should be stored where they will be available to run agricultural and transport equipment."

Quoting figures developed by Dr. Perry R. Stout, professor of soil science and chairman of the department of soils and plant nutrition of the University of California, Davis, Dr. Bellamy says that following an atomic attack, farm production will fall to about 29% of normal.

It is imperative, he points out, to stockpile a food reserve that will last two years or more—within walking distance of population centers and near drinking water. It is estimated that 750 pounds of supplementary dried food, which could be stored in a cube two feet, eight inches on a side, would be sufficient to keep an individual alive for the two-year period.

"We should not rely on reserves alone in stockpiling either fuel or food to sustain the population in the post-attack phase, but should strive for a balanced development of both categories," the UCLA biophysicist says. "There are far too many unforeseeable variables for us to risk being too little with too late."

He points out that stockpiling can be accomplished without undue economic strain. For many years we have been underwriting the costs of producing and storing many kinds of foods.

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