SN
EDUCATOR GUIDE
April 15, 2017

How Earth Got its Moon

Teacher Guide: Goodnight Moon

Class time: 30-50 minutes.

Purpose: To calculate the conditions that would have been required for the moon to "spin off" from Earth and end up with its current orbital angular momentum, and to consider the implications of those calculations for hypotheses about the moon's origin.

Background: The article "How Earth got its moon" discusses several hypotheses of how the moon formed. One idea in the 1800s was that perhaps Earth was spinning faster after it formed, and that material flew off Earth "like children tossed from an out-of-control merry-go-round" and then merged to form the moon. But how fast would Earth need to spin for this to be true? And would the moon end up rotating with the same angular momentum that it has today in this scenario? Angular momentum measures the tendency of a body that is rotating to keep rotating.

After students have reviewed the article "How Earth got its moon," pass out the accompanying student guide, Blackline Master 3. Students may benefit from access to calculators with scientific notation capability. Also, please discuss with your students that approximations are being made in the calculations for simplicity.

Depending on the level and background of your students, you may want to review the concepts below beforehand. The equations required to answer each question are included in the student guide. Feel free to select only the questions that relate to your class; however, once all of the given calculations are complete, students will be able to relate their answers to moon formation hypotheses. Another option is to give your students some of the equations and require them to remember or derive the rest.

This exercise uses the following skills:

- Scientific notation
- Converting between kilometers and meters
- Converting between days, hours, minutes and seconds
- Taking the square root of a number
- Calculating the square of a number
- Calculating the circumference of a circle
- Calculating the volume of a sphere
- Calculating density
- Calculating gravitational acceleration
- Calculating centrifugal (or centripetal) acceleration
- Calculating the velocity of a circular orbit
- Calculating angular momentum

Instructions: In this exercise, students will test the "spin off" hypothesis by calculating basic properties of Earth and the moon and then using them to calculate the orbital velocity of the moon and its angular momentum. They will use this information to calculate how much faster Earth would have to be spinning for material to be flung off it (the "breakaway velocity"). Finally, students will examine how that spin rate would affect Earth and decide whether the hypothesis is supported.

Here are some assumptions that are made in this exercise: The moon is in an approximately circular orbit around Earth. The Earth is much more massive than the moon, so a reasonable approximation is to treat the moon as orbiting around the center of Earth. (In fact, the orbit is slightly elliptical, and the Earth wobbles a bit since it is not infinitely more massive than the moon, but we are approximating here for simplicity.)

Here are some important values:

Mass of Earth = $m_{Earth} \approx 5.97 \times 10^{24}$ kg

Mass of the moon = $m_{moon} \approx 7.34 \times 10^{22}$ kg

Radius of the moon = $r_{moon} \approx 1740$ km

Radius of Earth = $r_{Earth} \approx 6370$ km

Radius of the moon's orbit = $r_{orbit} \approx 384{,}000$ km

Newton's gravitational constant = $G \approx 6.67 \times 10^{-11}$ kg

Questions:

1. In terms of radius, how small is the moon compared with Earth?

   $r_{moon} / r_{Earth} \approx 0.273 = 27.3\%$, The moon's radius is 27.3% that of Earth.

2. In terms of mass, how small is the moon compared with Earth?

   $m_{moon} / m_{Earth} \approx 0.0123 = 1.23\%$, The moon's mass is 1.23% that of Earth.

3. What is the volume of Earth?

   $V_{Earth} = (4/3)\,\pi\, r_{Earth}{}^3 \approx 1.08 \times 10^{21}\ m^3$

4. What is the average density of Earth? How does that compare with the density of water (1,000 kg/m³)?

   $m_{Earth}/V_{Earth} \approx 5.51 \times 10^3\ kg/m^3 \approx$ 5.51 times the density of water

5. What is the volume of the moon?

   $V_{moon} = (4/3)\,\pi\, r_{moon}{}^3 \approx 2.21 \times 10^{19}\ m^3$

6. What is the average density of the moon? How does that compare with the density of water (1,000 kg/m³)?

   $m_{moon}/V_{moon} \approx 3.33 \times 10^3\ kg/m^3 \approx$ 3.34 times the density of water

7. How does the average density of the moon compare with that of Earth?

   (moon density)/(Earth density) ≈ 0.604 ≈ 60.4%

8. Why do you think there is a density difference?

   Earth and the moon are composed of different elements or have different percentages of elementary components. For example, Earth has a much larger iron core than the moon does.

9. What is the radius of the moon's orbit using meters and scientific notation?

   $r_{orbit} \approx 3.84 \times 10^5$ km $= 3.84 \times 10^8$ m

10. What is the circumference of the moon's orbit around Earth?

    $2 \pi r_{orbit} \approx 2.41 \times 10^9$ m

11. For a stable circular orbit of the moon around Earth, the inward gravitational acceleration of the moon toward Earth ($G\, m_{Earth} / r_{orbit}^2$) must balance the outward centrifugal acceleration of the moon (v_{moon}^2/r_{orbit}). Assuming this is true, solve the equation for the orbital velocity of the moon.

    $v_{moon}^2 / r_{orbit} = G\, m_{Earth} / r_{orbit}^2$, *or*

    $v_{moon} = Sqrt\,(G\, m_{Earth} / r_{orbit})$

12. From the equation you solved above, what is the orbital velocity of the moon around Earth?

    $v_{moon} \approx 1.02 \times 10^3$ m/sec $= 1.02$ km/sec

13. One orbit of the moon around Earth takes $t_{orbit} \approx 27.3$ days. How many seconds is that?

    $t_{orbit} \approx 2.36 \times 10^6$ sec

14. Using the circumference of the moon's orbit and the time required for one orbit, what is the orbital velocity of the moon around Earth?

    $v_{moon} = 2 \pi r_{orbit} / t_{orbit} \approx 1.02 \times 10^3$ m/sec $= 1.02$ km/sec

15. Do your answers for questions 12 and 14 agree?

    They should match at least to about three significant figures.

16. What is the angular momentum of the moon, L_{moon}, due to its orbit around Earth?

    $L_{moon} = m_{moon}\, v_{orbit}\, r_{orbit}$

    $\approx 2.87 \times 10^{34}$ kg m^2 /sec

17. Angular momentum is conserved or stays constant unless an opposing force is present. If the moon had the same angular momentum, L_{moon}, that you just calculated but the entire mass of the moon were concentrated at Earth's surface r_{Earth} instead of the usual distance r_{orbit} from Earth, what would the moon's velocity, v, be? This velocity can represent the breakaway velocity of the moon.

    $v_{breakaway} = L_{moon} / (m_{moon}\, r_{Earth}) \approx 6.15 \times 10^4$ m/sec $= 61.5$ km/sec

18. What is the circumference of Earth?

    $2 \pi r_{Earth} \approx 4.00 \times 10^7 m = 40{,}000 km$

19. Earth makes one rotation in t_{Earth} = 24 hours. How many seconds is that?

    $t_{Earth} = 8.64 \times 10^4 sec$

20. What is the velocity of the surface of Earth at the equator, due to Earth's rotation?

    $v_{Earth} = (2 \pi r_{Earth}) / t_{Earth} \approx 463 m/sec$

21. If the moon were joined to Earth but just on the verge of breaking away from Earth's surface, how much faster would Earth have to be moving at the equator for the moon to have the correct angular momentum it ended up with?

    $v_{breakaway} / v_{Earth} \approx 133$ *times faster*

22. If the surface of Earth were moving fast enough at the equator for the breakaway moon to have that much angular momentum, how long would it take for Earth to make one complete rotation?

    $24 hours (v_{Earth} / v_{breakaway}) \approx 0.180 hours \approx 10.8 min$

23. What is the inward gravitational acceleration at Earth's surface?

    $1 gee = G m_{Earth} / r_{Earth}^2 \approx 9.81 m/sec^2$

24. At Earth's normal speed of rotation, what is the outward centrifugal acceleration at Earth's equator?

    $v_{Earth}^2/r_{Earth} \approx 0.0337 m/sec^2$

25. What is the net acceleration a person feels when standing at Earth's equator?

    $9.81 m/sec^2$ *inward* – $0.0337 m/sec^2$ *outward* ≈ $9.78 m/sec^2$ *inward* ≈ $0.997 g$

26. If the Earth were rotating fast enough for the moon to spin off it with its correct angular momentum, what would be the outward centrifugal acceleration at Earth's equator?

    $v_{breakaway}^2/r_{Earth} \approx 594 m/sec^2$

27. If Earth were rotating that fast, what would be the net acceleration a person would feel when standing at Earth's equator? What would happen to Earth?

    $594 m/sec^2$ *outward* – $9.81 m/sec^2$ *inward* ≈ $584 m/sec^2$ *outward* ≈ $59.5 g$ *outward*
    Everything would be flung off Earth, including the material making up Earth.

28. What do your calculations tell you about the validity of various hypotheses for the origin of the moon?

    Possible student responses: The moon could not have been formed by a fast-spinning young Earth because the planet would have had to be spinning so fast to shed the material for the moon that much more material would have been flung from it.

    Left on its own, the newly formed Earth would presumably not be rotating fast enough to spin off the moon.

One or more objects must have arrived carrying lots of angular momentum that ended up in the moon's orbit.

The entire moon could have come from elsewhere and entered into orbit around Earth with enough angular momentum, except it would likely be very difficult for Earth to capture the moon with its gravity.

If one or more objects hit Earth right in the center, they would not impart any angular momentum — it would give Earth a shove but not a spin.

If one or more objects came flying in, hit Earth off-center and then became the moon (possibly with some Earth material too), the moon could have the necessary angular momentum. Thus from our simplified calculations, this sort of scenario could explain the moon's origin and angular momentum.

Based on the average densities of Earth and the moon, Earth retained much of the dense iron core material from the proto-Earth, Theia or whatever else collided with it.

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IN HIGH SCHOOLS
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Questions:

1. In terms of radius, how small is the moon compared with Earth?

   $r_{moon} / r_{Earth} \approx$

2. In terms of mass, how small is the moon compared with Earth?

   $m_{moon} / m_{Earth} \approx$

3. What is the volume of Earth?

   $V_{Earth} = (4/3)\, \pi\, r_{Earth}^3 \approx$

4. What is the average density of Earth? How does that compare with the density of water (1,000 kg/m^3)?

   $m_{Earth}/V_{Earth} \approx$

5. What is the volume of the moon?

   $V_{moon} = (4/3)\, \pi\, r_{moon}^3 \approx$

6. What is the average density of the moon? How does that compare with the density of water (1,000 kg/m^3)?

   $m_{moon}/V_{moon} \approx$

7. How does the average density of the moon compare with that of Earth?

   (moon density)/(Earth density) ≈

8. Why do you think there is a density difference?

9. What is the radius of the moon's orbit using meters and scientific notation?

   $r_{orbit} \approx$

10. What is the circumference of the moon's orbit around Earth?

   $2\, \pi\, r_{orbit} \approx$

11. For a stable circular orbit of the moon around Earth, the inward gravitational acceleration of the moon toward Earth ($G\, m_{Earth} / r_{orbit}^2$) must balance the outward centrifugal acceleration of the moon (v_{moon}^2 / r_{orbit}). Assuming this is true, solve the equation for the orbital velocity of the moon.

12. From the equation you solved above, what is the orbital velocity of the moon around Earth?

   $v_{moon} \approx$

13. One orbit of the moon around Earth takes $t_{orbit} \approx 27.3$ days. How many seconds is that?

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$v_{breakaway} = L_{moon} / (m_{moon} r_{Earth}) \approx$

18. What is the circumference of Earth?

$2 \pi r_{Earth} \approx$

19. The Earth makes one rotation in t_{Earth} = 24 hours. How many seconds is that?

$t_{Earth} =$

20. What is the velocity of the surface of Earth at the equator, due to Earth's rotation?

$v_{Earth} = (2 \pi r_{Earth}) / t_{Earth} \approx$

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$v_{breakaway} / v_{Earth} \approx$

22. If the surface of Earth were moving fast enough at the equator for the breakaway moon to have that much angular momentum, how long would it take for Earth to make one complete rotation?

$24 \text{ hours} (v_{Earth} / v_{breakaway}) \approx$

23. What is the inward gravitational acceleration at Earth's surface?

$1 \text{ gee} = G m_{Earth} / r_{Earth}^2 \approx$

24. At Earth's normal speed of rotation, what is the outward centrifugal acceleration at Earth's equator?

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