SN EDUCATOR GUIDE

November 11, 2017

Neutron Star Crash Seen for First Time

Activity Guide for Students: The Pressure to Be a Star

Purpose: To understand the life cycle of stars and how stars produce various elements.

Procedural overview: Use basic algebra, physics and chemistry principles to estimate the conditions involved in stars and element formation.

Defined constants needed in the calculations:

- Speed of light = c = 3.00×10^8 m/sec
- Gravitational constant = G = 6.67×10^{-11} $\mathrm{m}^3/(\mathrm{kg}\bullet\mathrm{s}^2)$

1. Our sun is a fairly typical star, so we will do most of our calculations using it. The sun's average radius is $r = 6.96 \times 10^8$ m. Assuming that the sun is perfectly spherical (it is actually slightly oblate or squished), what is its volume?

2. For comparison, Earth's average radius is approximately 6.37×10^6 m. What is Earth's volume, and how many Earths would fit inside the sun?

3. The sun's mass is $M = 1.99 \times 10^{30}$ kg. What is the sun's average mass density, and how does that compare to the density of water (1000 $\mathrm{kg/m^3}$)?

4. Since the sun is neither contracting nor expanding, the gravitational force pulling matter inward toward the center of the sun must be equal to the outward force of gas pressure generated by the heat of fusion reactions, a condition known as hydrostatic balance. Detailed equations for hydrostatic balance may be found using calculus, but a simple estimate can be made with algebra. From Newton's law of gravity, the gravitational force pulling inward on matter of density ρ_{avg} near the outer edge of the sun would be $G\rho_{avg}M/r^2$. If the pressure at the center of the sun is P_c, the average pressure gradient between

the center of the sun and its edge is P_c/r. Setting those two forces equal would give $P_c/r = G\rho_{avg}M/r^2$, or $P_c = G\rho_{avg}M/r$. In reality, the pressure is much more concentrated in the center of the sun than that linear estimate suggests — roughly 100 times as concentrated — so a decent approximation is $P_c = 100\ G\rho_{avg} M/r$.

Using this approximation, what is the pressure at the center of the sun, and how does it compare with the pressure of Earth's atmosphere at sea level (1.01×10^5 Pa)? Solve the equation for a numerical answer, and also solve the equation using the appropriate units (1 Pa = kg/(m•s^2)).

5. Our assumption was that the pressure is concentrated 100-fold in the center compared with a simple linear estimate. That will compress the density to 100 times the average density. Calculate the density in the center of the sun:

6. Using the ideal gas law with a gas constant for monatomic hydrogen gas, R = 8315 J/(kg•K), the central pressure would be related to the central density and central temperature as $P_c = R\ \rho_c\ T_c$. But hydrogen in the sun is ionized, so its protons and its electrons each contribute that much pressure, and the total pressure is doubled, $P_c = 2\ R\ \rho_c\ T_c$.

What is the temperature at the center of the sun?

7. Normal room temperature is around 295 Kelvin (K). How many times hotter than room temperature is the center of the sun?

8. Based on the temperature thresholds at which these fusion reactions become significant, which of these reactions are most dominant in the sun?

$T_c > 8 \times 10^6$ K : proton-proton fusion to produce ^{4}He from protons

$T_c > 1.8 \times 10^7$ K : carbon-catalyzed (CNO) fusion to produce ^{4}He from protons

$T_c > 1 \times 10^8$ K : helium burning $^4He + ^4He \rightarrow ^8Be$ and $^8Be + ^4He \rightarrow ^{12}C + \gamma$

$T_c > 1 \times 10^8$ to 1×10^9 K: $^{12}C + ^4He \rightarrow ^{16}O + \gamma$ and $^{12}C + ^{12}C \rightarrow ^{23}Na + ^1H$

9. Fusing protons together forms helium-4 nuclei, also called α particles. The mass of one proton is $m_p = 1.6726 \times 10^{-27}$ kg. The mass of one α-particle is $m_\alpha = 6.6442 \times 10^{-27}$ kg. The total mass of four protons is larger than the mass of the helium-4 nucleus that the protons produced through fusion, so the excess mass is converted into energy. How much energy is produced by forming one α-particle?

10. The sun produces approximately 3.84×10^{26} W (J/sec) of energy. How many α-particles does it produce per second?

11. How much solar mass gets converted from hydrogen to helium per second?

12. If a neutron has a mass $m_n = 1.6749 \times 10^{-27}$ kg and an average effective radius of 9.41×10^{-16} m, what is its density?

13. If all the matter in the sun were compressed to form a neutron star with the same density as for the neutron you just calculated, how much more dense would that be than the sun's current average density?

14. If all the matter in the sun were compressed to form such a neutron star, what would its radius be?

15. A mass M must be compressed to a Schwarzschild radius of R_S to become a black hole. R_S is also the point at which light cannot escape. The escape velocity for an object from a given radius is that such that the object's positive kinetic energy is sufficient enough to cancel out the object's negative gravitational potential energy, $mv^2/2 - GMm/R = 0$, or $R = 2GM/v^2$. If light cannot escape, $v=c$, one finds the Schwarzschild radius $R_S = 2GM/c^2$. If the sun were compressed to form a black hole, what would its Schwarzschild radius be?

16. Out of all of these facts and numbers, what surprises you the most?

17. What other relevant characteristics or effects can you calculate using your knowledge of math and science?