SN
EDUCATOR GUIDE
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The Periodic Table Turns 150

Activity Guide for Students: The Periodic Table a Nuclear View

Directions for students: After reviewing the semi-empirical mass formula as a class, answer the questions below. These questions ask you to calculate nuclear binding energies and to understand the formula's implications for nuclear reactions and nuclear decays.

Background information:

The electrons orbiting the atomic nucleus participate in chemical reactions and govern the properties of elements. The protons and neutrons within the atomic nucleus participate in nuclear reactions and govern the nuclear stability and other nuclear properties of elements. The protons and neutrons within the nucleus follow rules that are similar to those that electrons follow.

Protons and neutrons are collectively termed nucleons. The number of protons is Z, the atomic number. The number of neutrons is N. The atomic mass A is Z + N, not counting tiny fractions removed for the binding energy (see explanation below).

Nucleons within a nucleus have a lower total energy than free nucleons. The amount by which the energy of nucleons in the nucleus is lower is the binding energy E_B, which is defined here as a positive value (electron binding energies are measured in eV and nuclear binding energies are measured in MeV).

Since $E = mc^2$, if the energy of the nucleus decreases by E_B, the mass of the nucleus decreases by E_B/c^2. Nuclear reactions that change the binding energy can convert small fractions of the mass of the nucleus into relatively large amounts of energy.

The semi-empirical mass formula treats the nucleus as a collection of a large number of nucleons, so it is important to recognize that the resulting number is not very accurate for small nuclei, basically nuclei of the first six elements or so.

A simple version of the semi-empirical mass formula for the nuclear binding energy is:

E_B (in MeV) = $16(A) - 18(A)^{(2/3)} - 0.71(Z)(Z - 1)/(A)^{(1/3)} - 24(N - Z)^2/(A)$ + pairing term + shell correction

The pairing term:

if both N and Z are even numbers = $34(A)^{(-3/4)}$

if N is odd and Z is even, or vice versa (meaning A is odd) = 0

if both N and Z are odd numbers = $-34(A)^{(-3/4)}$

The shell correction is approximately A/20 if N or Z is a stable number (2, 8, 20, 28, 50, 82 or 126) for a completely filled shell.

The binding energy per nucleon is E_B/A. The larger that number is, the more stable the average nucleon is in a nucleus.

Questions:

1. From examining the periodic table, what numbers of total electrons result in completely filled energy shells and therefore what are the most chemically stable elements?

2. There are also total numbers of protons that completely fill proton energy shells within the nucleus, and neutron totals that completely fill neutron energy shells. The stable numbers for protons or neutrons are different than those for electrons, since the forces within the nucleus are different. The numbers that result in stability, and high binding energy, for protons or neutrons are Z or N = 2, 8, 20, 28, 50, 82 and 126. Why do these number of protons and/or neutrons lead to stability? What elements and isotopes do the first three stable numbers (2, 8, 20) correspond to?

3. The most stable nuclei (highest value of E_B/A) are those located around iron and nickel on the periodic table. What is E_B/A for iron-56?

4. The semi-empirical mass formula is not designed to work well for very small nuclei, but for a rough estimate we can apply it to helium-4 and see what happens. What is E_B/A for helium-4?

5. The official value of E_B/A for helium-4 is approximately 7 MeV per nucleon. How close was your answer? Approximately how much energy would be produced by fusing four protons together to form helium-4, as occurs in stars? (Two of the protons become neutrons.)

6. What is E_B/A for uranium-235? If the nucleons from a uranium-235 nucleus could split to form iron-56 nuclei, what would be the average energy released per nucleon, and the total energy released by the uranium nucleus? (In practice, 235 nucleons might divide up into nuclei that are somewhat larger or smaller than iron-56, but those new nuclei would have binding energies close to what you have already calculated for iron-56, so we will just use that.)

7. What is E_B/A for a hypothetical new element with $Z = 120$ and $N = 180$? If the nucleons from an element 120 nucleus could split to form iron-56 nuclei, what would be the average energy release per nucleon, or the total energy released by element 120 nucleus? (In practice, 300 nucleons might divide up into nuclei that are somewhat larger or smaller than iron-56, but those new nuclei would have binding energies close to what you have already calculated for iron-56, so we will just use that.)

8. Based on your calculations for uranium and element 120, what term in the semi-empirical mass formula makes nuclei increasingly unstable as they get very large? What is the physical explanation for that effect?

9. Maximizing E_B for a given value of A gives the optimal fraction of nucleons that should be protons in order to achieve the greatest stability with respect to beta decay. The following equation, which gives the optimal fraction, can be derived from the semi-empirical mass formula:

$Z/A = 0.5/(1 + 0.0074(A)^{(2/3)})$

For A = 16, what optimal fraction and number of protons does the equation predict? (Round your answer to a whole number.) What element is that?

10. For A = 235, what optimal fraction and number of protons does the equation predict? (Round your answer to a whole number.) What element is that?

11. For A = 300, what optimal fraction and number of protons does the equation predict? (Round your answer to a whole number.) What element is that? From the "Long life" illustration in "Prospecting the periodic table," for the optimal number of protons calculated, what N value might be the most stable? How does that compare with the N value when A = 300 and Z is the value calculated?

12. In general, what does this equation predict about protons and neutrons in light elements? What is the physical reason for that?

13. In general, what does this equation predict about protons and neutrons in very heavy elements? What is the physical reason for that?

14. What implications does this equation have for the composition and radioactivity of nuclei produced by fission reactions?

15. What implications does this equation have for smashing smaller nuclei together as building blocks to create new very heavy elements?