Science News
IN HIGH SCHOOLS | EDUCATOR GUIDE

THE WORLD IN HDR/SHUTTERSTOCK

April 14, 2018

Stephen Hawking's Legacy Will Live On

SOCIETY FOR SCIENCE & THE PUBLIC

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

About the Issue

***Science News* article(s): "Stephen Hawking's legacy will live on"**

Readability score: 12.6

***Science News for Students* article(s):** "**Legendary physicist Stephen Hawking dies at 76**"

Readability score: 8.6

The article "Stephen Hawking's legacy will live on" gives a brief overview of the scientific work of Stephen Hawking, a famous physicist who died on March 14, 2018. Students can focus on information reported in the article, follow connections to earlier articles about Hawking's work and pursue cross-curricular connections in physics, biology and engineering. In a related activity, students can work individually or in small groups to derive equations and calculate a theoretical amount of Hawking radiation emitted by a black hole.

SN
EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

What's in this Guide?

Article-based observation: Questions focus on the work and legacy of cosmologist Stephen Hawking.

Quest through the archives: Use this short section to explore and compare other articles about Stephen Hawking's research as reported by *Science News* since 1924.

Cross-curricular discussion:

> **Physical Sciences** questions discuss Hawking radiation, mini black holes, singularities, the Big Bang and multiverses.
>
> **Biological Sciences** questions focus on amyotrophic lateral sclerosis (ALS) and its potential causes and treatments.
>
> **Engineering and Experimental Design** questions explore engineering approaches to aid ALS patients and what it might take for humans to colonize space.

Activity: Escaping from a Black Hole

> **Purpose:** To better understand black holes and Hawking radiation by deriving expressions and calculating theoretical data that relate to these phenomena.
>
> **Procedural overview:** Using the basic principles of general relativity and quantum mechanics, derive an expression, then calculate the Schwarzschild radius of a black hole and the temperature of the black hole's Hawking radiation. As an advanced extension, using calculus, derive the expression, then calculate the approximate amount of time required for a black hole emitting Hawking radiation to evaporate.
>
> **Approximate class time:** One class period.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Standards

Next Generation Science	Common Core ELA
Motion and Stability: Forces and Interactions: HS-PS2-1, HS-PS2-2, HS-PS2-4	Reading Informational Text (RI): 1, 2, 4, 5, 7
Energy: HS-PS3-1, HS-PS3-2, HS-PS3-4, HS-PS3-5	Writing (W): 1, 2, 3, 4, 6, 7, 8, 9
Waves and Their Applications in Technologies for Information Transfer: HS-PS4-1, HS-PS4-3, HS-PS4-4	Speaking and Listening (SL): 1, 2, 4, 5, 6
From Molecules to Organisms: Structures and Processes: HS-LS1-1, HS-LS1-2	Reading for Literacy in Science and Technical Subjects (RST): 1, 2, 3, 4, 5, 7, 8, 9
Earth's Place in the Universe: HS-ESS-1-2, HS-ESS-1-4	Writing Literacy in History/Social Studies and Science and Technical Subjects (WHST): 1, 2, 4, 7, 8, 9
Engineering Design: HS-ETS1-1, HS-ETS1-2, HS-ETS1-3	

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Article-Based Observation: Q&A

These questions are based on the article "Stephen Hawking's legacy will live on."

1. Where did Stephen Hawking study and work?

Possible student response: Hawking received his Ph.D. from the University of Cambridge in 1965 and then studied cosmology there for the rest of his life.

2. What was Hawking's most famous discovery and when did it occur?

Possible student response: In 1974, Hawking reported that black holes are not entirely black. Rather, black holes emit a faint glow of particles. Today, that glow is commonly called Hawking radiation.

3. How does Hawking radiation occur? Has it ever been observed?

Possible student response: According to quantum mechanics, pairs of particles and antiparticles pervade all of space. Each virtual particle and its antiparticle partner annihilate one another almost as soon as the pair appears. However, if a particle-antiparticle pair appears just outside of a black hole and one of the particles falls into the black hole, the surviving member of the pair can escape. Black holes emit these newly-single particles, which produces a faint glow. Hawking radiation has never been directly observed.

4. Why does Hawking radiation cause a paradox according to quantum mechanical principles? What is one possible solution to the paradox proposed by Hawking and his colleagues? Why is this paradox difficult for scientists to solve?

Possible student response: Hawking radiation suggests that black holes can eventually evaporate and disappear. Any information within those black holes would also disappear. But quantum mechanics says that information cannot be destroyed. In 2016, Hawking proposed that black holes might have "soft hair," low-energy particles that would retain information about what fell into a black hole. Other scientists have proposed potential solutions. This topic is especially difficult since it combines both quantum physics (the physics of very small things) and general relativity (which describes gravity). A theory of quantum gravity that combines both of those aspects of physics might provide answers, but we do not currently have an accepted theory of quantum gravity.

5. What were some of Hawking's other research accomplishments?

Possible student response: Hawking studied points in which the fabric of spacetime is infinitely curved, mini black holes that may have formed in the early universe and multiverses (universes parallel to our own.)

6. How did Hawking help to popularize science?

Possible student response: Hawking wrote books that made abstract physics clear and understandable for the public. His most famous book, *A Brief History of Time*, and others inspired new generations of scientists and science enthusiasts.

7. What disease did Hawking have? What effects did the disease have on him?

Possible student response: Hawking had amyotrophic lateral sclerosis (ALS), a degenerative disease that caused him to gradually lose control of his body. He used a wheelchair to move and a computer voice synthesizer to speak.

8. What questions do you still have after reading the article?

Possible student response: According to quantum physics, why can information never be destroyed? What would it take to experimentally observe and confirm the existence of Hawking radiation? What exactly did Hawking find regarding mini black holes and multiverses? What causes amyotrophic lateral sclerosis, and how could ALS be treated or possibly cured?

9. Write a poem to commemorate Stephen Hawking's work as described by the article. Name the type of poem that you select to write.

Possible student response: The following is a Haiku:

Luminous black holes
Embracing quantum physics
Stephen Hawking lives

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Article-Based Observation: Q

Directions: Read the article "Stephen Hawking's legacy will live on" and then answer these questions:

1. Where did Stephen Hawking study and work?

2. What was Hawking's most famous discovery and when did it occur?

3. How does Hawking radiation occur? Has it ever been observed?

4. Why does Hawking radiation cause a paradox according to quantum mechanical principles? What is one possible solution to the paradox proposed by Hawking and his colleagues? Why is this paradox difficult for scientists to solve?

5. What were some of Hawking's other research accomplishments?

6. How did Hawking help to popularize science?

7. What disease did Hawking have? What effects did the disease have on him?

8. What questions do you still have after reading the article?

9. Write a poem to commemorate Stephen Hawking's work as described by the article. Name the type of poem that you select to write.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Quest Through the Archives: Q&A

1. What is the earliest article that you can find about Hawking and black holes?

Possible student response: The *Science News* article "Black hole power," published 4/13/1974, explores mini black holes that could be relatively close to the Earth. Stephen Hawking had only recently proposed that there could be mini black holes, far smaller than conventional stellar black holes. Mini black holes might have been created in the high-density early moments of the universe after the Big Bang. Hawking encouraged astronomers to search for signs that such mini black holes might exist within our solar system. Physicists Lowell Wood, Thomas Weaver and John Nuckolls took Hawking's suggestion even further. These physicists, of Lawrence Livermore National Laboratory in Livermore, Calif., had been working on ways to produce controlled thermonuclear fusion reactions with lasers. The researchers suggested that a mini black hole's intense gravitational field may be able to compress and heat hydrogen to ignite energy-producing fusion reactions. Most of the resulting helium and energy would escape from the black hole. That escaping energy could be harvested and converted into electricity.

2. Do black holes destroy or preserve information? Find an article that explains Hawking's proposed solution to the black hole information paradox.

Possible student response: The *Science News* article "Hawking proposes solution to black hole problem," published 8/26/2015, describes how information might be preserved in black holes. Quantum physics says that quantum information cannot be destroyed. But any particles falling into a black hole are hidden from the rest of the universe, until eventually the black hole evaporates due to Hawking radiation. That apparent destruction of information is termed the black hole information paradox. To resolve the paradox, Hawking and colleagues proposed that as objects pass through an event horizon (the point at which light can no longer escape) and enter a black hole, the objects leave behind detailed images or holograms that retain the objects' information. Such holograms take the form of light orbiting the black hole at the event horizon, never falling into the black hole, but never able to escape either. Much more work is needed to investigate this idea.

3. Find a *Science News for Students* Cool Jobs article that mentions Stephen Hawking. Why is he included in the article and what is the article about?

Possible student response: The *Science News for Students* article "Cool Jobs: Keeping TV science honest," published 9/8/2016, gives the inside scoop on what it takes to make popular TV shows about science and medicine realistic. The article highlights scientists who are hired to work on the scripts, acting and even the sets of shows such as *Bones*, *Chicago Med* and *The Big Bang Theory*. David Saltzberg, a physics professor at the University of California, Los Angeles, is responsible for making sure science used in *The Big Bang Theory* is accurate, even when the science is used in the satirical sense. Stephen Hawking is mentioned in the article because he made a guest appearance on *The Big Bang Theory*.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Quest Through the Archives: Q

Directions: After reading the article "Stephen Hawking's legacy will live on," log in to your *Science News* in High Schools account and use the Search page to answer these questions. Make sure you adjust the filters to include articles written before 1999, if the question requires you to do so.

1. What is the earliest article that you can find about Stephen Hawking and black holes?

2. Do black holes destroy or preserve information? Find an article that explains Hawking's proposed solution to the black hole information paradox.

3. Find a *Science News for Students* Cool Jobs article that mentions Stephen Hawking. Why is he included in the article and what is the article about?

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Cross-Curricular Discussion: Q&A

Directions: After students have had a chance to review the article "Stephen Hawking's legacy will live on," lead a classroom discussion based on the questions that follow. See the "Related articles" section to find supplemental articles for students.

PHYSICAL SCIENCES

Discussion questions:

1. What are quantum fluctuations?

Just as there is a Heisenberg uncertainty relation between momentum and position (a small amount of uncertainty, given by Planck's constant, must always remain in one or the other or both), there is also a Heisenberg uncertainty relation between energy and time. There is always a small amount of uncertainty in energy and/or time. In other words, the shorter the lifetime of a particle, the greater the uncertainty of the energy of the particle. A quantum fluctuation is a temporary variation in the energy in empty space due to the uncertainty principle. This violation of energy conservation, even if only for a very short time, gives rise to the brief existence of pairs of particles and antiparticles (electrons and positrons, pairs of photons or other particle pairs) that spontaneously appear in existence from empty space. Almost as soon as these particle pairs exist, they annihilate each other and disappear. Such particle pairs are "virtual" — the particles and antiparticles do not officially have enough energy to exist indefinitely.

2. What is Hawking radiation?

Throughout all of space, even empty space, virtual particle-antiparticle pairs continually appear and then annihilate each other. If a pair appears near the event horizon of a black hole, one member may fall into the black hole leaving its partner to escape. The black hole emits the escaping particle or antiparticle as radiation is emitted and the mass of the black hole is reduced accordingly.

3. What is a mini black hole?

Most black holes are thought to be created by the collapse of large stars, resulting in black holes with large masses. Stephen Hawking predicted that the very dense state of the early universe immediately after the Big Bang might have created mini black holes with far smaller masses. Stellar black holes are far from Earth, but mini black holes might be floating around anywhere in the universe, possibly even within our solar system.

4. What are multiverses?

Multiverses are parallel universes that exist in addition to our universe. Some parallel universes might be very similar to ours — parallel universes could have branched off from our universe when an event offered alternate outcomes. Other parallel universes might be very different than ours — fundamental constants *such as the speed of light and strength of gravity might be different, or the laws of physics could be entirely different. Depending on the constants and the laws of physics, some universes could be more likely to give rise to organized matter and ultimately intelligent life, and other universes could be less likely to do so. Perhaps the Big Bang was when our universe branched off from another universe, or perhaps it was even when two universes collided to produce our universe. Parallel universes have yet to be observed.*

Extension prompts:

5. How might you experimentally observe and confirm the existence of Hawking radiation?

To produce Hawking radiation, the starting virtual particle pairs require a sufficiently strong gravitational field at the event horizon of a black hole to rip apart. The rate of particles ripping apart is relatively small. Therefore, observers would need to be relatively close, closer than is possible without getting pulled into the black hole, to observe Hawking radiation. Scientists do not know of any black holes in our solar system that can be directly observed. However, scientists may have overlooked a mini black hole somewhere in our solar system. A mini black hole might evaporate in a relatively large burst of Hawking radiation, so perhaps such large but rare events could be detected with gamma ray telescopes or other instruments that monitor deep space. We might also consider artificial methods of attempting to create a black hole, or we might consider means of sending probes beyond our solar system to visit black holes.

6. How can the production of particle-antiparticle pairs be observed?

The electric field near an atomic nucleus (especially one with high atomic number) can briefly separate virtual electron-positron pairs, although the pairs can still recombine and disappear. Yet if enough energy is provided, those pairs can become real and continue to exist. High-energy photons (such as gamma rays) passing near atomic nuclei generate real electron-positron pairs. This process, called pair production, has been observed and is one of the main methods by which gamma rays are absorbed by matter.

7. What is a gravitational singularity?

Space and time may be regarded as a single flexible sheet in four (or more) dimensions, called spacetime. Gravitational fields are the bending and stretching of spacetime, and gravitational waves are vibrations of spacetime. A singularity is a point at which the curvature of spacetime becomes infinite, or at which the gravitational field becomes infinitely strong. The center of a black hole is a singularity. The initial state of the universe prior to the Big Bang may have been a singularity.

BIOLOGICAL SCIENCES

Discussion questions:

1. Stephen Hawking was diagnosed with amytrophic lateral sclerosis (ALS) when he was 21. What is ALS?

ALS is a disease that destroys the motor neurons, or nerve cells that control muscles. Early symptoms include muscle stiffness, weakness, twitching, cramps and atrophy. As the disease progresses, the patient usually loses the ability to move, speak, swallow and eventually breathe. Patients typically die from respiratory failure within about four years of symptom onset. Some famous people with ALS include: Stephen Hawking (1942-2018), a cosmologist who lived a remarkable 55 years with the disease after his diagnosis in 1963, and Lou Gehrig (1903-1941), who played baseball for the New York Yankees in the 1920s and '30s. ALS is still frequently called Lou Gehrig's disease.

Extension prompts:

2. What are some potential causes of ALS?

The causes of ALS are not well understood. Some ALS cases appear to be due to inherited genetic mutations, such as mutations in the superoxide dismutase (SOD) gene. This gene's protein product normally helps to eliminate damaging oxygen radicals in neurons and other cells. More specifically, SOD is an enzyme (produced by the corresponding gene) that converts O^{2-} *superoxide radicals into normal* O_2 *or* H_2O_2 *(then the catalase enzyme converts* H_2O_2 *into* H_2O *and* O_2*). The enzyme therefore gets rid of damaging superoxide radicals that are by-products of oxidative phosphorylation in cells' energy-producing organelles called mitochondria. If some ALS patients have a mutation that makes SOD ineffective or less effective, patients' cells would presumably have more superoxide radicals and more oxidative damage. Why that affects primarily motor neurons and not lots of other cell types is not currently clear. Other ALS cases appear to be linked to repetitive head injuries such as those sustained by some soldiers and athletes. Many ALS cases have no obvious cause.*

As ALS progresses, motor neurons in the brain's motor cortex (muscle control region), brain stem and spinal cord die. Those motor neurons tend to contain abnormal clumps of proteins, including ubiquitin (a protein that usually tags other proteins to be safely destroyed within a cell) and SOD. Just as what triggers the death of motor neurons in ALS remains unknown, it is also currently unknown why other neurons are relatively unaffected by the disease.

3. What are some current and potential treatments for ALS?

Riluzole is a small drug molecule that blocks some sodium ion channels, NMDA receptors and other motor neuron receptors associated with ALS. But the drug currently prolongs life by only a few months.

Potential future treatments may include improved drugs that bind to relevant receptors or ion channels on motor neurons, fight inflammation, reduce protein clumping and target some cells' protein-destroying systems. Various gene therapy approaches could potentially affect any gene products known or suspected to be involved in ALS, including sodium ion channels, NMDA receptors and SOD.

ENGINEERING AND EXPERIMENTAL DESIGN

Discussion questions:

1. In what ways can engineering be used to assist ALS patients?

Motorized wheelchairs, robotic arms and other technology can assist ALS patients with mobility. As ALS progresses, patients need mechanical assistance with bodily functions ranging from breathing to swallowing. Such devices are currently external, but future versions could potentially be implanted. The most challenging problem is obtaining output signals from patients with advanced ALS, who might only be able to slightly move one facial muscle (like Stephen Hawking). If devices could reliably convert brainwaves to output signals, for example, ALS patients may be able to communicate more information more rapidly, which might improve patients' quality of life.

Extension prompts:

2. Stephen Hawking believed that it was important to establish human colonies in space. He expressed that human life on Earth could be eliminated by a virus, war, asteroid or some other catastrophe. How could humans establish colonies in space?

Humans have lived on the International Space Station and other space stations in low Earth orbit for short periods of time over the last 40 years or more. However, those stations must be supplied by frequent cargo rockets from Earth. And the stations' lack of artificial gravity causes bone and muscle loss as well as other physiological changes that limit the duration of human visits. To be more viable, a space station would need to be large enough to sustain a significant population of people, rotate to generate artificial gravity and have a self-sustaining environment with enough plants and essential materials onboard to provide humans with resources to live, such as breathable air. Living on another planet or a moon would provide access to raw materials, much more room and at least some gravity, but would also likely require initial resupplying. The moon and Mars are the most suitable locations for human colonization.

3. What do you think is the likelihood that human life on Earth will be wiped out by an infectious disease, war, environmental change, an asteroid or other factors? Explain your reasoning.

Student answers will vary. Students can self-select different groups that argue for specific catastrophes that may impact humanity, or argue that no catastrophe will, and summarize their reasoning for their peers.

4. Do you agree with Stephen Hawking that there is a moral imperative for humankind to spread to other parts of space to avoid extinction? Would the universe be better off if humans remain on Earth? Or do you have other ideas?

This is another opportunity for students to self-select into groups based on their opinions and to explain the reasoning for such opinions to the rest of the class. Groups could be in favor of spreading humankind to space, protecting space from humankind or staying on Earth but protecting ourselves from extinction.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Cross-Curricular Discussion: Q

Directions: The following list of discussion questions is provided to help you take notes, brainstorm ideas and test your thinking in order to be more actively engaged in class discussions related to this article. All questions in this section are related to topics covered in "Stephen Hawking's legacy will live on."

PHYSICAL SCIENCES

Discussion questions:

1. What are quantum fluctuations?

2. What is Hawking radiation?

3. What is a mini black hole?

4. What are multiverses?

Extension prompts:

5. How might you experimentally observe and confirm the existence of Hawking radiation?

6. How can the production of particle-antiparticle pairs be observed?

7. What is a gravitational singularity?

BIOLOGICAL SCIENCES

Discussion questions:

1. Stephen Hawking was diagnosed with amytrophic lateral sclerosis (ALS) when he was 21. What is ALS?

Extension prompts:

2. What are some potential causes of ALS?

3. What are some current and potential treatments for ALS?

ENGINEERING AND EXPERIMENTAL DESIGN

Discussion questions:

1. In what ways can engineering be used to assist ALS patients?

Extension prompts:

2. Stephen Hawking believed that it was important to establish human colonies in space. He expressed that human life on Earth could be eliminated by a virus, war, asteroid or some other catastrophe. How could humans establish colonies in space?

3. What do you think is the likelihood that human life on Earth will be wiped out by an infectious disease, war, environmental change, an asteroid or other factors? Explain your reasoning.

4. Do you agree with Stephen Hawking that there is a moral imperative for humankind to spread to other parts of space to avoid extinction? Would the universe be better off if humans remain on Earth? Or do you have other ideas?

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Activity Guide for Teachers: Escaping from a Black Hole

Purpose: To better understand black holes and Hawking radiation by deriving expressions and calculating theoretical data that relate to these phenomena.

Procedural overview: Using the basic principles of general relativity and quantum mechanics, derive an expression, then calculate the Schwarzschild radius of a black hole and the temperature of the black hole's Hawking radiation. As an advanced extension, using calculus, derive the expression, then calculate the approximate amount of time required for a black hole emitting Hawking radiation to evaporate.

Approximate class time: One class period.

Supplies:

- Student handout: Escaping from a Black Hole
- Scientific calculators

Directions for teachers: Students can work through this activity in class, where you can offer help if they do not understand something. If class time is limited and the students are sufficiently advanced, they could do this activity as homework instead. Students could work individually, although it may be helpful for them to work in groups to discuss their reasoning along the way. Suggested student responses for correctly derived equations and calculated answers are given in italics below.

Directions for students: In Part A, you can derive an expression for the size of a black hole (the Schwarzschild radius of its event horizon, the classical point of no return for everything including light) with a given mass. Note that even though you will use a classical physics derivation and not special or general relativity, your answer is still an excellent approximation.

In Part B, you can derive an expression, then calculate the temperature of Hawking radiation emitted by a black hole. Matter or energy may escape from black holes in the form of Hawking radiation. According to quantum mechanics, pairs of virtual particles and antiparticles continually appear throughout space and then promptly annihilate each other and disappear. Hawking radiation occurs when such a particle-antiparticle pair appear just outside of a black hole's event horizon. Stephen Hawking theorized that when one member of the pair crosses the event horizon, becoming trapped in the black hole, the remaining particle (or antiparticle) is emitted by the black hole.

A hot object emits photons with a spectrum of frequencies governed by how hot the object is. As an object heats up, it emits infrared photons, followed by red light, yellow light, blue light, ultraviolet light and so forth. This is called black body radiation because the color of the radiation is due to the temperature of the object, not the object's inherent color. The emission of Hawking radiation makes a black hole appear to be radiating like a black body at a certain temperature.

In Part C, you can calculate the time it would take for a black hole to evaporate. Theoretically, black holes that emit Hawking radiation also lose mass and can eventually disappear.

Part A. Calculating the Schwarzschild radius of a black hole

1. Newton's gravitational constant is $G \approx 6.67\times10^{-11}$ meters^{3}kilograms^{-1}second^{-2}. If a small object of mass m (in kg) is separated from a large object of mass M (in kg) by a distance of r (in m), what is the gravitational potential energy (in Joules) of the small object? This potential energy is relative to being infinitely far away from the large object, $r=\infty$, with zero gravitational potential energy.

$$P.E. = -G M m/r$$

2. If the small object is moving away from the large object with a velocity v (in m/sec), what is the kinetic energy of the small object (in J)?

$$K.E. = m v^2/2$$

3. What is the total energy of the small object when it is a distance r from the large object and has a velocity v?

$$E = K.E. + P.E. = (m v^2/2) - (G M m/r)$$

4. If the small object coasts away from the large object until it slows to a stop, and it just barely escapes from the large object's gravity ($r=\infty$) without falling back, what is the total energy of the small object?

$$E = K.E. + P.E. = 0$$

5. By equating the small object's total energy before and after escaping from the large object, what escape velocity v_{escape} must the small object have when it is a distance r from the large object in order to eventually just barely escape?

$$v_{escape} = (2 G M/r)^{1/2}$$

6. A black hole has such strong gravity that even light (traveling at velocity $c \approx 3.00\times10^8$ m/sec) cannot escape if it gets too close. The radius of a black hole's event horizon, the point at which light can no longer escape, is called the Schwarzschild radius R_S.. Everything within that radius appears black to an outside observer. If the escape velocity becomes the speed of light, $v_{escape} = c$, at the event horizon, $r=R_S$, what is the Schwarzschild radius?

$$R_S = 2 G M/c^2$$

Our sun's mass is $M_S \approx 1.99\times10^{30}$ kg, so the black hole's mass M can be expressed in multiples of solar masses:

$$M = (M/M_S)\ 1.99\times10^{30} \text{ kg}$$

7. Plugging in that expression for mass as well as the numbers for G and c, how would you express the Schwarzschild radius in multiples of solar masses?

$$R_S \approx (2)(6.67\times10^{-11})(1.99\times10^{30})/(9.00\times10^{16}) \quad (M/M_S)\ m$$

$$\approx 2950\ (M/M_S)\ m \quad = \quad 2950\ (M/M_S)\ m$$

Part B. Calculating the temperature of Hawking radiation from a black hole

Planck's constant, $h \approx 6.626\times10^{-34}$ J sec, governs the size of quantum effects. The Heisenberg uncertainty relation between measurements of momentum (Δp) and position (Δx) is:

$$(\Delta p)\ (\Delta x) \approx (h/4\pi)$$

8. Because photons may be emitted from any part of the event horizon, whose dimensions are described by the Schwarzschild radius R_s or diameter 2 R_s, there is an uncertainty $(\Delta x) \sim 2R_s$ for the initial position of the photons. What is the corresponding uncertainty in the momentum of emitted photons, Δp?

$$\Delta p \approx h/(8\ \pi\ R_S)\ =\ (h\ c^2)/(16\ \pi\ G\ M)$$

This uncertainty in momentum corresponds to uncertainty in the photon energy Δp c. That energy can be characterized in terms of thermal energy fluctuations k_BT at a temperature T (with $k_B \approx 1.38\times10^{-23}$ Joules/Kelvin defined as Boltzmann's constant):

$$\Delta p\ c \approx k_B\ T$$

9. Equating these two expressions for Δp and solving for T, what is the temperature corresponding to Hawking radiation?

$$T \approx (h\ c^3)/(16\ \pi\ k_B\ G\ M)$$

10. Look up Stephen Hawking's derivation of the temperature of a black hole's event horizon. How close did your answer come to Hawking's?

Stephen Hawking's much more rigorous derivation of the temperature of a black hole's event horizon gave the answer:

$$T = (h\ c^3)/(16\ \pi^2\ k_B\ G\ M)$$

Our estimate was only off by a factor of π, which is not too shabby.

11. Plugging in the numbers for the constants, how would you express Hawking's value for the temperature of a black hole (in Kelvin) in terms of solar masses (relative to our sun)?

$$T \approx [(6.626\times10^{-34})(3.00\times10^{8})^3]/[(16\ \pi^2)(1.38\times10^{-23})(6.67\times10^{-11})(\ 1.99\times10^{30})]\ (M_s/M)\ K$$

$$\approx (M_s/M)\ 6.18\times10^{-8}\ K$$

Note how cold this is! The surface of a black hole the mass of our sun is barely warm. Such a black hole barely emits particles. Much less massive black holes are hotter and emit more particles.

Part C. Advanced extension (using calculus): Calculating the time for a black hole to evaporate by emitting Hawking radiation

The Stefan-Boltzmann constant is:

$$\sigma_{SB} = \frac{(2\,\pi^5 * k_B^4)}{(15 * h^3 * c^2)} \approx 5.67 \times 10^{-8}\,\frac{\text{watt}}{m^2 * K^4}$$

A stationary black hole has an energy E = M∗c². Due to Hawking radiation, a black hole loses energy at a rate (d/dt is the derivative with respect to time, or the change per time):

$$\left(\frac{d}{dt}\right) * (M * c^2) = -(4 * \pi * R^2)(\sigma_{SB} * T^4)$$

12. What is the equation if you plug in the expressions for the Schwarzschild radius, Hawking's temperature for the black hole and the equation (not the number) for the Stefan-Boltzmann constant?

(d/dt) $(M c^2) = -(h c^6)/(30720\,\pi^2 G^2 M^2)$

13. If you plug in numbers and express the black hole's mass in terms of solar masses, what is the power of Hawking radiation from a black hole?

(d/dt) $(M c^2) = -(M_s/M)^2\ 9.04\times10^{-29}\ W$

That is a tiny amount of radiation from a stellar-mass black hole and therefore a tiny energy loss from the total energy of the black hole. But black holes with smaller masses have higher temperatures and lose energy more rapidly than more massive black holes. Mini black holes should completely evaporate in one final explosion of Hawking radiation, unless other presently unknown quantum gravity effects intervene.

You can integrate your equation from question 12 above to find the time $t_{evaporation}$ (t_{evap}) for a black hole of initial mass M_0 to evaporate via Hawking radiation.

Below is all the calculus you need to know. Ignoring various multiplicative factors, your equation has the following starting point, which you can then separate out variables and integrate to solve:

$$\frac{dM}{dt} = -\frac{1}{M^2}$$

$$dt = -M^2 * dM$$

$$\int_0^{t_{evap}} dt = -\int_{M_0}^{0} M^2 * dM$$

$$\int_0^{t_{evap}} dt = \int_0^{M_0} M^2 * dM$$

$$t_{evap} = \frac{M_0^3}{3}$$

14. Use your equation from question 12 and the above calculus trick (but with all the relevant constants included this time) to find the time it takes for a black hole of initial mass M_0 to evaporate via Hawking radiation:

$$t_{evap} = (10240\ \pi^2\ G^2)(h\ c^4)\ M_0^3$$

15. Use your answer to question 14 and plug in the constants to find the evaporation time (in seconds or years) for a black hole with an initial mass expressed in multiples of solar masses or in kg:

$$t_{evap} \approx (M_0/M_S)^3\ 6.60\times10^{74}\ sec$$

$$\approx (M_0/M_S)^3\ 2.09\times10^{67}\ years$$

$$\approx (M_0/10^{11}\ kg)^3\ 10\ billion\ years$$

16. Have black holes evaporated since the Big Bang, which occurred 13.8 billion years ago? Do a sample calculation and explain your thought process.

Hawking theorized that mini black holes could have been formed soon after the Big Bang, when the universe was denser. Most stellar-mass black holes may have formed later, as large stars ultimately burned out one by one. Thus a small black hole of ~ 10^{11} kg or less created early in the 13.8-billion-year history of the universe would have evaporated by now. But more typical stellar-mass black holes would require far longer to evaporate than the universe's predicted remaining lifetime of about 5 billion years.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Activity Guide for Students: Escaping from a Black Hole

Purpose: To better understand black holes and Hawking radiation by deriving expressions and calculating theoretical data that relate to these phenomena.

Procedural overview: Using the basic principles of general relativity and quantum mechanics, derive an expression, then calculate the Schwarzschild radius of a black hole and the temperature of the black hole's Hawking radiation. As an advanced extension, using calculus, derive the expression, then calculate the approximate amount of time required for a black hole emitting Hawking radiation to evaporate.

Approximate class time: One class period.

Supplies:
- Student handout: Escaping from a Black Hole
- Scientific calculators

Directions: In Part A, you can derive an expression for the size of a black hole (the Schwarzschild radius of its event horizon, the classical point of no return for everything including light) with a given mass. Note that even though you will use a classical physics derivation and not special or general relativity, your answer is still an excellent approximation.

In Part B, you can derive an expression, then calculate the temperature of Hawking radiation emitted by a black hole. Matter or energy may escape from black holes in the form of Hawking radiation. According to quantum mechanics, pairs of virtual particles and antiparticles continually appear throughout space and then promptly annihilate each other and disappear. Hawking radiation occurs when such a particle-antiparticle pair appear just outside of a black hole's event horizon. Stephen Hawking theorized that when one member of the pair crosses the event horizon, becoming trapped in the black hole, the remaining particle (or antiparticle) is emitted by the black hole.

A hot object emits photons with a spectrum of frequencies governed by how hot the object is. As an object heats up, it emits infrared photons, followed by red light, yellow light, blue light, ultraviolet light and so forth. This is called black body radiation because the color of the radiation is due to the temperature of the object, not the object's inherent color. The emission of Hawking radiation makes a black hole appear to be radiating like a black body at a certain temperature.

In Part C, you can calculate the time it would take for a black hole to evaporate. Theoretically, black holes that emit Hawking radiation also lose mass and can eventually disappear.

Part A. Calculating the Schwarzschild radius of a black hole

1. Newton's gravitational constant is $G \approx 6.67 \times 10^{-11}$ meters^{3}kilograms^{-1}second^{-2}. If a small object of mass m (in kg) is separated from a large object of mass M (in kg) by a distance of r (in m), what is the gravitational potential energy (in Joules) of the small object? This potential energy is relative to being infinitely far away from the large object, $r=\infty$, with zero gravitational potential energy.

2. If the small object is moving away from the large object with a velocity v (in m/sec), what is the kinetic energy of the small object (in J)?

3. What is the total energy of the small object when it is a distance r from the large object and has a velocity v?

4. If the small object coasts away from the large object until it slows to a stop, and it just barely escapes from the large object's gravity ($r=\infty$) without falling back, what is the total energy of the small object?

5. By equating the small object's total energy before and after escaping from the large object, what escape velocity v_{escape} must the small object have when it is a distance r from the large object in order to eventually just barely escape?

6. A black hole has such strong gravity that even light (traveling at velocity $c \approx 3.00\times10^8$ m/sec) cannot escape if it gets too close. The radius of a black hole's event horizon, the point at which light can no longer escape, is called the Schwarzschild radius R_s.. Everything within that radius appears black to an outside observer. If the escape velocity becomes the speed of light, $v_{escape} = c$, at the event horizon, $r=R_s$, what is the Schwarzschild radius?

Our sun's mass is $M_s \approx 1.99\times10^{30}$ kg, so the black hole's mass M can be expressed in multiples of solar masses:

$$M = (M/M_s)\ 1.99\times10^{30} \text{ kg}$$

7. Plugging in that expression for mass as well as the numbers for G and c, how would you express the Schwarzschild radius in multiples of solar masses?

Part B. Calculating the temperature of Hawking radiation from a black hole

Planck's constant, $h \approx 6.626\times10^{-34}$ J sec, governs the size of quantum effects. The Heisenberg uncertainty relation between measurements of momentum (Δp) and position (Δx) is:

$$(\Delta p)\ (\Delta x) \approx (h/4\pi)$$

8. Because photons may be emitted from any part of the event horizon, whose dimensions are described by the Schwarzschild radius R_s or diameter 2 R_s, there is an uncertainty $(\Delta x) \sim 2R_s$ for the initial position of the photons. What is the corresponding uncertainty in the momentum of emitted photons, Δp?

This uncertainty in momentum corresponds to uncertainty in the photon energy $\Delta p\ c$. That energy can be characterized in terms of thermal energy fluctuations $k_B T$ at a temperature T (with $k_B \approx 1.38\times10^{-23}$ J/ K defined as Boltzmann's constant):

$$\Delta p\ c \approx k_B\ T$$

9. Equating these two expressions for Δp and solving for T, what is the temperature corresponding to Hawking radiation?

10. Look up Stephen Hawking's derivation of the temperature of a black hole's event horizon. How close did your answer come to Hawking's?

11. Plugging in the numbers for the constants, how would you express Hawking's value for the temperature of a black hole (in Kelvin) in terms of solar masses (relative to our sun)?

Part C. Advanced extension (using calculus): Calculating the time for a black hole to evaporate by emitting Hawking radiation

The Stefan-Boltzmann constant is:

$$\sigma_{SB} = \frac{(2\,\pi^5 * k_B^4)}{(15 * h^3 * c^2)} \approx 5.67 \times 10^{-8}\ \frac{\text{watt}}{m^2 * K^4}$$

A stationary black hole has an energy E = M∗c². Due to Hawking radiation, a black hole loses energy at a rate (d/dt is the derivative with respect to time, or the change per time):

$$\left(\frac{d}{dt}\right) * (M * c^2) = -(4 * \pi * R^2)(\sigma_{SB} * T^4)$$

12. What is the equation if you plug in the expressions for the Schwarzschild radius, Hawking's temperature for the black hole and the equation (not the number) for the Stefan-Boltzmann constant?

13. If you plug in numbers and express the black hole's mass in terms of solar masses, what is the power of Hawking radiation from a black hole?

That is a tiny amount of radiation from a stellar-mass black hole and therefore a tiny energy loss from the total energy of the black hole. But black holes with smaller masses have higher temperatures and lose energy more rapidly than more massive black holes. Mini black holes should completely evaporate in one final explosion of Hawking radiation, unless other presently unknown quantum gravity effects intervene.

You can integrate your equation from question 12 above to find the time $t_{evaporation}$ (t_{evap}) for a black hole of initial mass M_0 to evaporate via Hawking radiation.

Below is all the calculus you need to know. Ignoring various multiplicative factors, your equation has the following starting point, which you can then separate out variables and integrate to solve:

$$\frac{dM}{dt} = -\frac{1}{M^2}$$

$$dt = -M^2 * dM$$

$$\int_0^{t_{evap}} dt = -\int_{M_0}^{0} M^2 * dM$$

$$\int_0^{t_{evap}} dt = \int_0^{M_0} M^2 * dM$$

$$t_{evap} = \frac{M_0^3}{3}$$

14. Use your equation from question 12 and the above calculus trick (but with all the relevant constants included this time) to find the time it takes for a black hole of initial mass M_0 to evaporate via Hawking radiation:

15. Use your answer to question 14 and plug in the constants to find the evaporation time (in seconds or years) for a black hole with an initial mass expressed in multiples of solar masses or in kg:

16. Have black holes evaporated since the Big Bang, which occurred 13.8 billion years ago? Do a sample calculation and explain your thought process.

SN EDUCATOR GUIDE

April 14, 2018

Stephen Hawking's Legacy Will Live On

Other Related Articles

***Science News*:**
"Renowned physicist Stephen Hawking dies at 76"

"Superconductors may shed light on the black hole information paradox"

"Hawking proposes solution to black hole problem"

***Science News for Students*:**
"Disabilities don't stop these experts in science and tech"
Readability score: 6.9

"Stephen Hawking says his group has solved a black hole puzzle"
Readability score: 7.8

"Escape from a lab-built black hole"
Readability score: 7.9

SOCIETY FOR SCIENCE & THE PUBLIC

© Society for Science & the Public 2000–2018. All rights reserved.