SN
IN HIGH SCHOOLS
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How Earth Got its Moon

Student Guide: Goodnight Moon

The article "How Earth got its moon" discusses several hypotheses of how the moon formed. One idea in the 1800s was that perhaps Earth was spinning faster after it formed, and that material flew off Earth "like children tossed from an out-of-control merry-go-round" and then merged to form the moon. But how fast would Earth need to spin for this to be true? And would the moon end up rotating with the same angular momentum that it has today in this scenario? Angular momentum measures the tendency of a body that is rotating to keep rotating.

Here are some assumptions that are made in this exercise: The moon is in an approximately circular orbit around Earth. The Earth is much more massive than the moon, so a reasonable approximation is to treat the moon as orbiting around the center of Earth. (In fact, the orbit is slightly elliptical, and the Earth wobbles a bit since it is not infinitely more massive than the moon, but we are approximating here for simplicity.)

Here are some important values:

Mass of Earth = $m_{Earth} \approx 5.97 \times 10^{24}$ kg

Mass of the moon = $m_{moon} \approx 7.34 \times 10^{22}$ kg

Radius of the moon = $r_{moon} \approx 1740$ km

Radius of Earth = $r_{Earth} \approx 6370$ km

Radius of the moon's orbit = $r_{orbit} \approx 384{,}000$ km

Newton's gravitational constant = $G \approx 6.67 \times 10^{-11}$ kg

Questions:

1. In terms of radius, how small is the moon compared with Earth?

   $r_{moon} / r_{Earth} \approx$

2. In terms of mass, how small is the moon compared with Earth?

   $m_{moon} / m_{Earth} \approx$

3. What is the volume of Earth?

   $V_{Earth} = (4/3)\, \pi\, r_{Earth}^3 \approx$

4. What is the average density of Earth? How does that compare with the density of water (1,000 kg/m^3)?

   $m_{Earth}/V_{Earth} \approx$

5. What is the volume of the moon?

   $V_{moon} = (4/3)\,\pi\, r_{moon}^3 \approx$

6. What is the average density of the moon? How does that compare with the density of water (1,000 kg/m^3)?

   $m_{moon}/V_{moon} \approx$

7. How does the average density of the moon compare with that of Earth?

   (moon density)/(Earth density) $\approx$

8. Why do you think there is a density difference?

9. What is the radius of the moon's orbit using meters and scientific notation?

   $r_{orbit} \approx$

10. What is the circumference of the moon's orbit around Earth?

    $2\,\pi\, r_{orbit} \approx$

11. For a stable circular orbit of the moon around Earth, the inward gravitational acceleration of the moon toward Earth ($G\, m_{Earth} / r_{orbit}^2$) must balance the outward centrifugal acceleration of the moon (v_{moon}^2 / r_{orbit}). Assuming this is true, solve the equation for the orbital velocity of the moon.

12. From the equation you solved above, what is the orbital velocity of the moon around Earth?

    $v_{moon} \approx$

13. One orbit of the moon around Earth takes $t_{orbit} \approx 27.3$ days. How many seconds is that?

    $t_{orbit} \approx$

14. Using the circumference of the moon's orbit and the time required for one orbit, what is the orbital velocity of the moon around Earth?

    $v_{moon} = 2\,\pi\, r_{orbit} / t_{orbit} \approx$

15. Do your answers for questions 12 and 14 agree?

16. What is the angular momentum of the moon, L_{moon}, due to its orbit around Earth?

$L_{moon} = m_{moon}\, v_{orbit}\, r_{orbit} \approx$

17. Angular momentum is conserved or stays constant unless an opposing force is present. If the moon had the same angular momentum, L_{moon}, that you just calculated but the entire mass of the moon were concentrated at Earth's surface r_{Earth} instead of the usual distance r_{orbit} from Earth, what would the moon's velocity, v, be? This velocity can represent the breakaway velocity of the moon.

$v_{breakaway} = L_{moon} / (m_{moon}\, r_{Earth}) \approx$

18. What is the circumference of Earth?

$2\,\pi\, r_{Earth} \approx$

19. The Earth makes one rotation in t_{Earth} = 24 hours. How many seconds is that?

$t_{Earth} =$

20. What is the velocity of the surface of Earth at the equator, due to Earth's rotation?

$v_{Earth} = (2\,\pi\, r_{Earth}) / t_{Earth} \approx$

21. If the moon were joined to Earth but just on the verge of breaking away from Earth's surface, how much faster would Earth have to be moving at the equator for the moon to have the correct angular momentum it ended up with?

$v_{breakaway} / v_{Earth} \approx$

22. If the surface of Earth were moving fast enough at the equator for the breakaway moon to have that much angular momentum, how long would it take for Earth to make one complete rotation?

$24\ hours\ (v_{Earth} / v_{breakaway}) \approx$

23. What is the inward gravitational acceleration at Earth's surface?

$1\ gee = G\, m_{Earth} / r_{Earth}^2 \approx$

24. At Earth's normal speed of rotation, what is the outward centrifugal acceleration at Earth's equator?

$v_{Earth}^2 / r_{Earth} \approx$

25. What is the net acceleration a person feels when standing at Earth's equator?

26. If the Earth were rotating fast enough for the moon to spin off it with its correct angular momentum, what would be the outward centrifugal acceleration at Earth's equator?

$v_{breakaway}^2 / r_{Earth} \approx$

27. If Earth were rotating that fast, what would be the net acceleration a person would feel when standing at Earth's equator? What would happen to Earth?

28. What do your calculations tell you about the validity of various hypotheses for the origin of the moon? Explain.